

Sec 3.3

Try as a Class

Vector Representation by Directed Line Segments

Let \mathbf{u} be represented by the directed line segment from $P(0, 0)$ to $Q(3, 2)$, and let \mathbf{v} be represented by the directed line segment from $R(1, 2)$ to $S(4, 4)$, as shown in Figure 3.17. Show that \mathbf{u} and \mathbf{v} are equivalent.

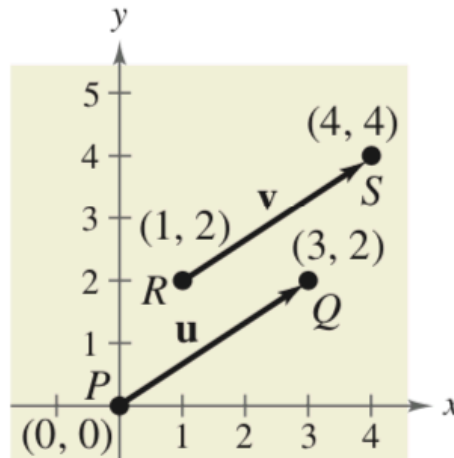


FIGURE 3.17

Solution

Solution

From the Distance Formula, it follows that \overrightarrow{PQ} and \overrightarrow{RS} have the *same magnitude*.

$$\|\overrightarrow{PQ}\| = \sqrt{(3 - 0)^2 + (2 - 0)^2} = \sqrt{13} \quad \|\overrightarrow{RS}\| = \sqrt{(4 - 1)^2 + (4 - 2)^2} = \sqrt{13}$$

Moreover, both line segments have the *same direction* because they are both directed toward the upper right on lines having a slope of

$$\frac{4 - 2}{4 - 1} = \frac{2 - 0}{3 - 0} = \frac{2}{3}.$$

Because \overrightarrow{PQ} and \overrightarrow{RS} have the same magnitude and direction, \mathbf{u} and \mathbf{v} are equivalent.

Component Form of a Vector

The component form of the vector with initial point $P(p_1, p_2)$ and terminal point $Q(q_1, q_2)$ is given by

$$\overrightarrow{PQ} = \langle q_1 - p_1, q_2 - p_2 \rangle = \langle v_1, v_2 \rangle = \mathbf{v}.$$

The **magnitude** (or length) of \mathbf{v} is given by

$$\|\mathbf{v}\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} = \sqrt{v_1^2 + v_2^2}.$$

If $\|\mathbf{v}\| = 1$, \mathbf{v} is a **unit vector**. Moreover, $\|\mathbf{v}\| = 0$ if and only if \mathbf{v} is the zero vector $\mathbf{0}$.

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Finding the Component Form of a Vector

Find the component form and magnitude of the vector \mathbf{v} that has initial point $(4, -7)$ and terminal point $(-1, 5)$.

Solution

Algebraic Solution

Let

$$P(4, -7) = (p_1, p_2)$$

and

$$Q(-1, 5) = (q_1, q_2).$$

Then, the components of $\mathbf{v} = \langle v_1, v_2 \rangle$ are

$$v_1 = q_1 - p_1 = -1 - 4 = -5$$

$$v_2 = q_2 - p_2 = 5 - (-7) = 12.$$

So, $\mathbf{v} = \langle -5, 12 \rangle$ and the magnitude of \mathbf{v} is

$$\begin{aligned}\|\mathbf{v}\| &= \sqrt{(-5)^2 + 12^2} \\ &= \sqrt{169} = 13.\end{aligned}$$

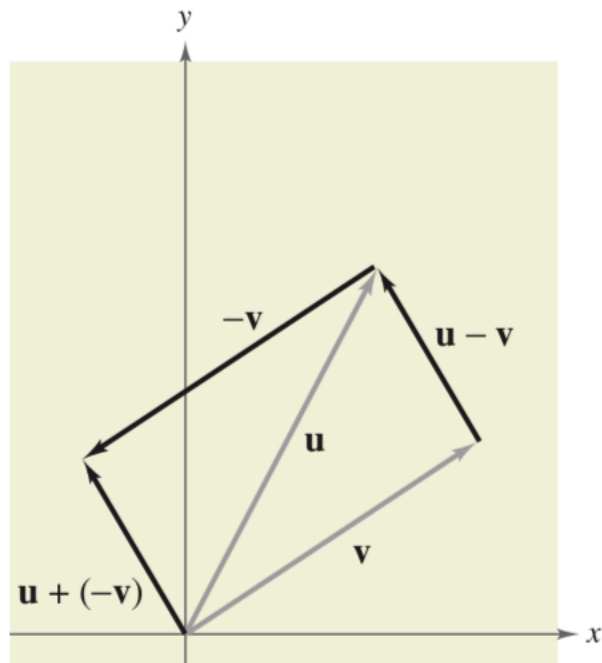
Definitions of Vector Addition and Scalar Multiplication

Let $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ be vectors and let k be a scalar (a real number). Then the *sum* of \mathbf{u} and \mathbf{v} is the vector

$$\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2 \rangle \quad \text{Sum}$$

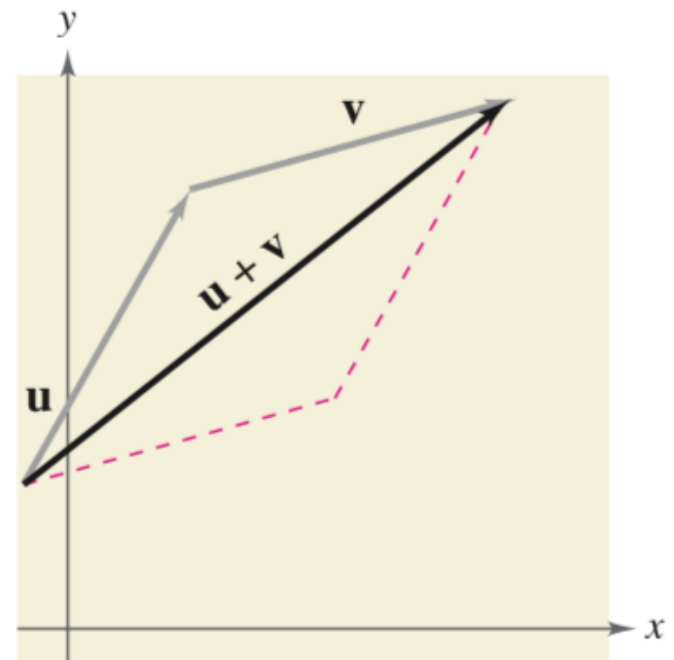
and the *scalar multiple* of k times \mathbf{u} is the vector

$$k\mathbf{u} = k\langle u_1, u_2 \rangle = \langle ku_1, ku_2 \rangle. \quad \text{Scalar multiple}$$



$$\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v})$$

FIGURE 3.21



Example

Vector Operations

Let $\mathbf{v} = \langle -2, 5 \rangle$ and $\mathbf{w} = \langle 3, 4 \rangle$, and find each of the following vectors.

- a. $2\mathbf{v}$ b. $\mathbf{w} - \mathbf{v}$ c. $\mathbf{v} + 2\mathbf{w}$

Solution

- a. Because $\mathbf{v} = \langle -2, 5 \rangle$, you have

$$\begin{aligned} 2\mathbf{v} &= 2\langle -2, 5 \rangle \\ &= \langle 2(-2), 2(5) \rangle \\ &= \langle -4, 10 \rangle. \end{aligned}$$

A sketch of $2\mathbf{v}$ is shown in Figure 3.22.

- b. The difference of \mathbf{w} and \mathbf{v} is

$$\begin{aligned} \mathbf{w} - \mathbf{v} &= \langle 3, 4 \rangle - \langle -2, 5 \rangle \\ &= \langle 3 - (-2), 4 - 5 \rangle \\ &= \langle 5, -1 \rangle. \end{aligned}$$

A sketch of $\mathbf{w} - \mathbf{v}$ is shown in Figure 3.23. Note that the figure shows the vector difference $\mathbf{w} - \mathbf{v}$ as the sum $\mathbf{w} + (-\mathbf{v})$.

- c. The sum of \mathbf{v} and $2\mathbf{w}$ is

$$\begin{aligned} \mathbf{v} + 2\mathbf{w} &= \langle -2, 5 \rangle + 2\langle 3, 4 \rangle \\ &= \langle -2, 5 \rangle + \langle 2(3), 2(4) \rangle \\ &= \langle -2, 5 \rangle + \langle 6, 8 \rangle \\ &= \langle -2 + 6, 5 + 8 \rangle \\ &= \langle 4, 13 \rangle. \end{aligned}$$

A sketch of $\mathbf{v} + 2\mathbf{w}$ is shown in Figure 3.24.

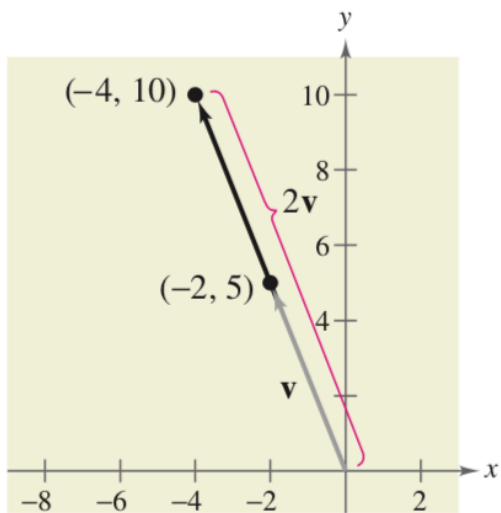


FIGURE 3.22

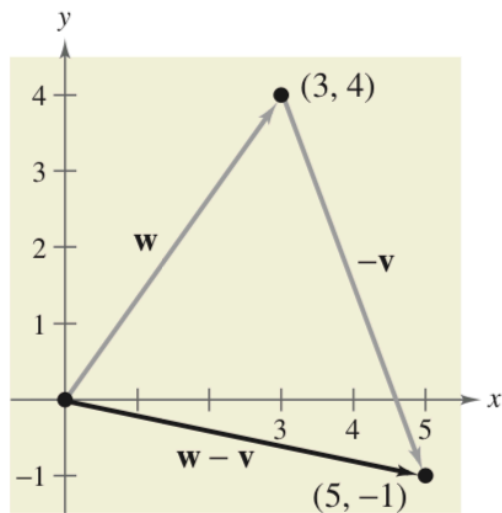


FIGURE 3.23

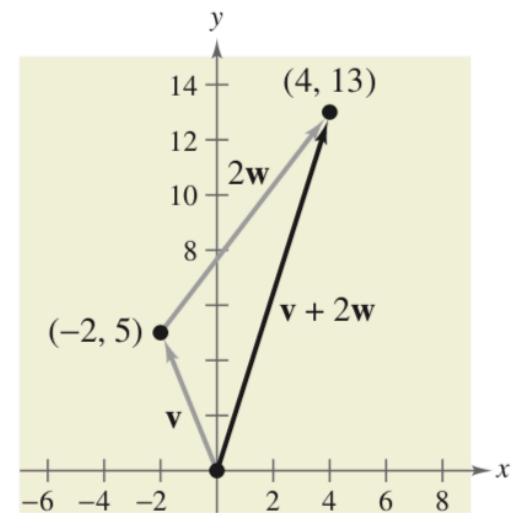


FIGURE 3.24

Properties of Vector Addition and Scalar Multiplication

Let \mathbf{u} , \mathbf{v} , and \mathbf{w} be vectors and let c and d be scalars. Then the following properties are true.

1. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$

3. $\mathbf{u} + \mathbf{0} = \mathbf{u}$

4. $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$

5. $c(d\mathbf{u}) = (cd)\mathbf{u}$

6. $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$

7. $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$

8. $1(\mathbf{u}) = \mathbf{u}, \quad 0(\mathbf{u}) = \mathbf{0}$

9. $\|c\mathbf{v}\| = |c| \|\mathbf{v}\|$

Unit Vectors

$$\mathbf{u} = \text{unit vector} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \left(\frac{1}{\|\mathbf{v}\|} \right) \mathbf{v}.$$

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Find a unit vector in the direction of $\mathbf{v} = \langle -2, 5 \rangle$ and verify that the result has a magnitude of 1.

Solution

Solution

The unit vector in the direction of \mathbf{v} is

$$\begin{aligned}\frac{\mathbf{v}}{\|\mathbf{v}\|} &= \frac{\langle -2, 5 \rangle}{\sqrt{(-2)^2 + (5)^2}} \\ &= \frac{1}{\sqrt{29}} \langle -2, 5 \rangle \\ &= \left\langle \frac{-2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \right\rangle.\end{aligned}$$

This vector has a magnitude of 1 because

$$\sqrt{\left(\frac{-2}{\sqrt{29}}\right)^2 + \left(\frac{5}{\sqrt{29}}\right)^2} = \sqrt{\frac{4}{29} + \frac{25}{29}} = \sqrt{\frac{29}{29}} = 1.$$

Example

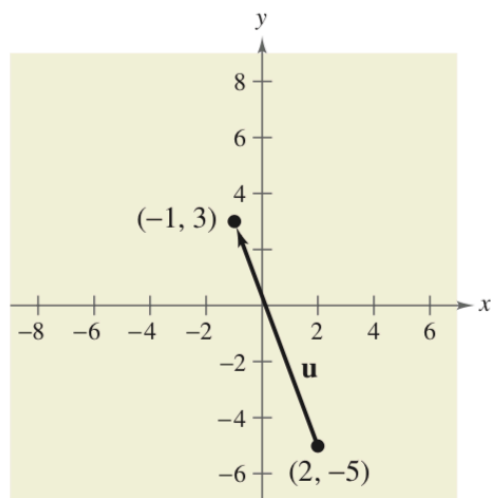


FIGURE 3.26

Writing a Linear Combination of Unit Vectors

Let \mathbf{u} be the vector with initial point $(2, -5)$ and terminal point $(-1, 3)$. Write \mathbf{u} as a linear combination of the standard unit vectors \mathbf{i} and \mathbf{j} .

Solution

Begin by writing the component form of the vector \mathbf{u} .

$$\begin{aligned}\mathbf{u} &= \langle -1 - 2, 3 - (-5) \rangle \\ &= \langle -3, 8 \rangle \\ &= -3\mathbf{i} + 8\mathbf{j}\end{aligned}$$

This result is shown graphically in Figure 3.26.

CHECKPoint → Now try Exercise 53.

Try as a Class

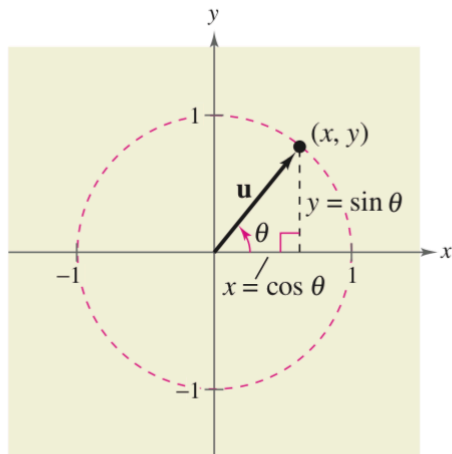
Let $\mathbf{u} = -3\mathbf{i} + 8\mathbf{j}$ and let $\mathbf{v} = 2\mathbf{i} - \mathbf{j}$. Find $2\mathbf{u} - 3\mathbf{v}$.

Solution

Solution

You could solve this problem by converting \mathbf{u} and \mathbf{v} to component form. This, however, is not necessary. It is just as easy to perform the operations in unit vector form.

$$\begin{aligned} 2\mathbf{u} - 3\mathbf{v} &= 2(-3\mathbf{i} + 8\mathbf{j}) - 3(2\mathbf{i} - \mathbf{j}) \\ &= -6\mathbf{i} + 16\mathbf{j} - 6\mathbf{i} + 3\mathbf{j} \\ &= -12\mathbf{i} + 19\mathbf{j} \end{aligned}$$



$$\|\mathbf{u}\| = 1$$

FIGURE 3.27

Direction Angles

If \mathbf{u} is a *unit vector* such that θ is the angle (measured counterclockwise) from the positive x -axis to \mathbf{u} , the terminal point of \mathbf{u} lies on the unit circle and you have

$$\mathbf{u} = \langle x, y \rangle = \langle \cos \theta, \sin \theta \rangle = (\cos \theta)\mathbf{i} + (\sin \theta)\mathbf{j}$$

as shown in Figure 3.27. The angle θ is the **direction angle** of the vector \mathbf{u} .

Suppose that \mathbf{u} is a unit vector with direction angle θ . If $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$ is any vector that makes an angle θ with the positive x -axis, it has the same direction as \mathbf{u} and you can write

$$\begin{aligned}\mathbf{v} &= \|\mathbf{v}\| \langle \cos \theta, \sin \theta \rangle \\ &= \|\mathbf{v}\| (\cos \theta)\mathbf{i} + \|\mathbf{v}\| (\sin \theta)\mathbf{j}.\end{aligned}$$

Because $\mathbf{v} = a\mathbf{i} + b\mathbf{j} = \|\mathbf{v}\|(\cos \theta)\mathbf{i} + \|\mathbf{v}\|(\sin \theta)\mathbf{j}$, it follows that the direction angle θ for \mathbf{v} is determined from

$$\begin{aligned}\tan \theta &= \frac{\sin \theta}{\cos \theta} && \text{Quotient identity} \\ &= \frac{\|\mathbf{v}\| \sin \theta}{\|\mathbf{v}\| \cos \theta} && \text{Multiply numerator and denominator by } \|\mathbf{v}\|. \\ &= \frac{b}{a}. && \text{Simplify.}\end{aligned}$$

Example

Finding Direction Angles of Vectors

Find the direction angle of each vector.

a. $\mathbf{u} = 3\mathbf{i} + 3\mathbf{j}$

b. $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$

Solution

a. The direction angle is

$$\tan \theta = \frac{b}{a} = \frac{3}{3} = 1.$$

So, $\theta = 45^\circ$, as shown in Figure 3.28.

b. The direction angle is

$$\tan \theta = \frac{b}{a} = \frac{-4}{3}.$$

Moreover, because $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$ lies in Quadrant IV, θ lies in Quadrant IV and its reference angle is

$$\theta = \left| \arctan\left(-\frac{4}{3}\right) \right| \approx |-53.13^\circ| = 53.13^\circ.$$

So, it follows that $\theta \approx 360^\circ - 53.13^\circ = 306.87^\circ$, as shown in Figure 3.29.

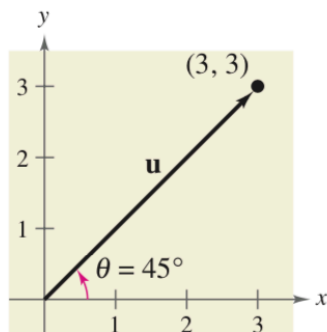


FIGURE 3.28

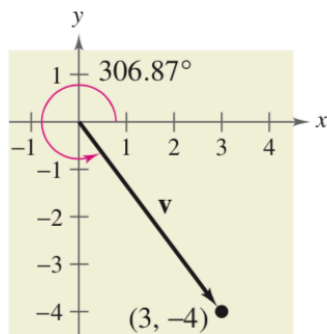


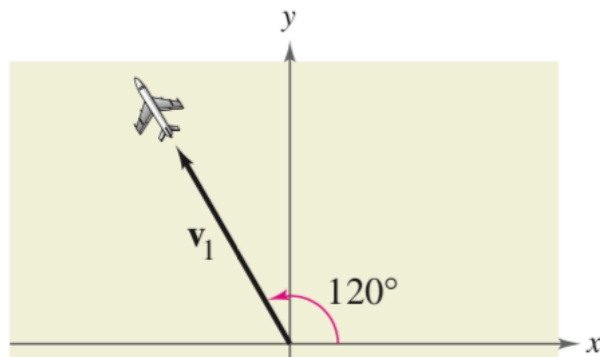
FIGURE 3.29

CHECKPoint Now try Exercise 63.

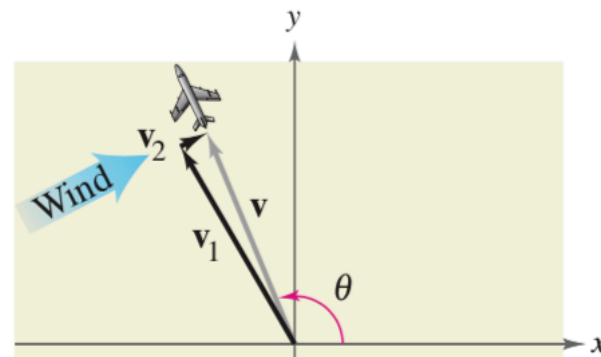
Try as a Class

Using Vectors to Find Speed and Direction

An airplane is traveling at a speed of 500 miles per hour with a bearing of 330° at a fixed altitude with a negligible wind velocity as shown in Figure 3.32(a). When the airplane reaches a certain point, it encounters a wind with a velocity of 70 miles per hour in the direction $N 45^\circ E$, as shown in Figure 3.32(b). What are the resultant speed and direction of the airplane?



(a)



(b)

FIGURE 3.32

Solution

Solution

Using Figure 3.32, the velocity of the airplane (alone) is

$$\begin{aligned}\mathbf{v}_1 &= 500\langle \cos 120^\circ, \sin 120^\circ \rangle \\ &= \langle -250, 250\sqrt{3} \rangle\end{aligned}$$

and the velocity of the wind is

$$\begin{aligned}\mathbf{v}_2 &= 70\langle \cos 45^\circ, \sin 45^\circ \rangle \\ &= \langle 35\sqrt{2}, 35\sqrt{2} \rangle.\end{aligned}$$

So, the velocity of the airplane (in the wind) is

$$\begin{aligned}\mathbf{v} &= \mathbf{v}_1 + \mathbf{v}_2 \\ &= \langle -250 + 35\sqrt{2}, 250\sqrt{3} + 35\sqrt{2} \rangle \\ &\approx \langle -200.5, 482.5 \rangle\end{aligned}$$

and the resultant speed of the airplane is

$$\begin{aligned}\|\mathbf{v}\| &\approx \sqrt{(-200.5)^2 + (482.5)^2} \\ &\approx 522.5 \text{ miles per hour.}\end{aligned}$$

Finally, if θ is the direction angle of the flight path, you have

$$\begin{aligned}\tan \theta &\approx \frac{482.5}{-200.5} \\ &\approx -2.4065\end{aligned}$$

which implies that

$$\theta \approx 180^\circ + \arctan(-2.4065) \approx 180^\circ - 67.4^\circ = 112.6^\circ.$$

So, the true direction of the airplane is approximately

$$270^\circ + (180^\circ - 112.6^\circ) = 337.4^\circ.$$

Sec 3.4

Definition of the Dot Product

The **dot product** of $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ is

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2.$$

Properties of the Dot Product

Let \mathbf{u} , \mathbf{v} , and \mathbf{w} be vectors in the plane or in space and let c be a scalar.

1. $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$

2. $\mathbf{0} \cdot \mathbf{v} = 0$

3. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$

4. $\mathbf{v} \cdot \mathbf{v} = \|\mathbf{v}\|^2$

5. $c(\mathbf{u} \cdot \mathbf{v}) = c\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot c\mathbf{v}$

Try as a Class



Finding Dot Products

Find each dot product.

a. $\langle 4, 5 \rangle \cdot \langle 2, 3 \rangle$ **b.** $\langle 2, -1 \rangle \cdot \langle 1, 2 \rangle$ **c.** $\langle 0, 3 \rangle \cdot \langle 4, -2 \rangle$

Solution

Solution

$$\begin{aligned}\text{a. } \langle 4, 5 \rangle \cdot \langle 2, 3 \rangle &= 4(2) + 5(3) \\ &= 8 + 15 \\ &= 23\end{aligned}$$

$$\begin{aligned}\text{b. } \langle 2, -1 \rangle \cdot \langle 1, 2 \rangle &= 2(1) + (-1)(2) \\ &= 2 - 2 = 0\end{aligned}$$

$$\begin{aligned}\text{c. } \langle 0, 3 \rangle \cdot \langle 4, -2 \rangle &= 0(4) + 3(-2) \\ &= 0 - 6 = -6\end{aligned}$$

Try as a Class

Using Properties of Dot Products

Let $\mathbf{u} = \langle -1, 3 \rangle$, $\mathbf{v} = \langle 2, -4 \rangle$, and $\mathbf{w} = \langle 1, -2 \rangle$. Find each dot product.

a. $(\mathbf{u} \cdot \mathbf{v})\mathbf{w}$

b. $\mathbf{u} \cdot 2\mathbf{v}$

Solution

Solution

Begin by finding the dot product of \mathbf{u} and \mathbf{v} .

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle -1, 3 \rangle \cdot \langle 2, -4 \rangle \\ &= (-1)(2) + 3(-4) \\ &= -14\end{aligned}$$

$$\begin{aligned}\text{a. } (\mathbf{u} \cdot \mathbf{v})\mathbf{w} &= -14\langle 1, -2 \rangle \\ &= \langle -14, 28 \rangle\end{aligned}$$

$$\begin{aligned}\text{b. } \mathbf{u} \cdot 2\mathbf{v} &= 2(\mathbf{u} \cdot \mathbf{v}) \\ &= 2(-14) \\ &= -28\end{aligned}$$

Notice that the product in part (a) is a vector, whereas the product in part (b) is a scalar. Can you see why?

Try as a Class



Dot Product and Magnitude

The dot product of \mathbf{u} with itself is 5. What is the magnitude of \mathbf{u} ?

Solution

Solution

Because $\|\mathbf{u}\|^2 = \mathbf{u} \cdot \mathbf{u}$ and $\mathbf{u} \cdot \mathbf{u} = 5$, it follows that

$$\begin{aligned}\|\mathbf{u}\| &= \sqrt{\mathbf{u} \cdot \mathbf{u}} \\ &= \sqrt{5}.\end{aligned}$$

Angle Between Two Vectors

If θ is the angle between two nonzero vectors \mathbf{u} and \mathbf{v} , then

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}.$$

Try as a Class

Finding the Angle Between Two Vectors

Find the angle θ between $\mathbf{u} = \langle 4, 3 \rangle$ and $\mathbf{v} = \langle 3, 5 \rangle$.

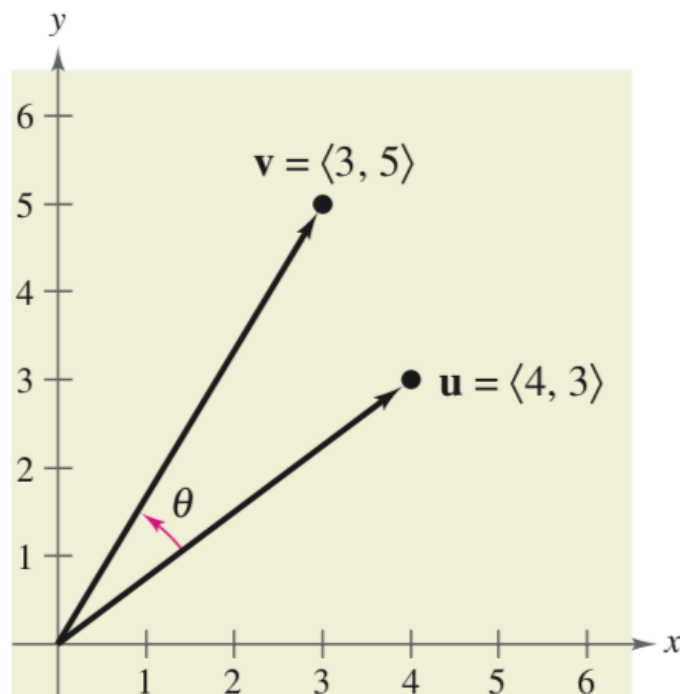


FIGURE 3.34

Solution

Solution

The two vectors and θ are shown in Figure 3.34.

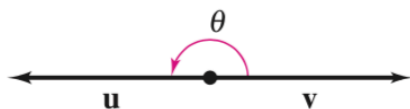
$$\begin{aligned}\cos \theta &= \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} \\ &= \frac{\langle 4, 3 \rangle \cdot \langle 3, 5 \rangle}{\|\langle 4, 3 \rangle\| \|\langle 3, 5 \rangle\|} \\ &= \frac{27}{5\sqrt{34}}\end{aligned}$$

This implies that the angle between the two vectors is

$$\theta = \arccos \frac{27}{5\sqrt{34}} \approx 22.2^\circ.$$

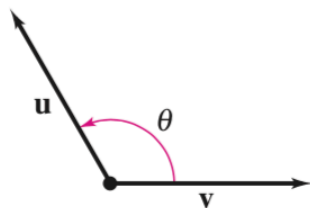
$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta$$

Alternative form of dot product

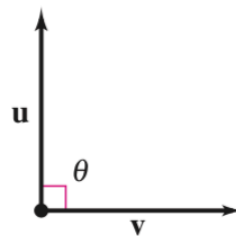


$\theta = \pi$
 $\cos \theta = -1$
 Opposite Direction

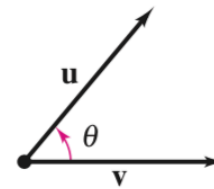
FIGURE 3.35



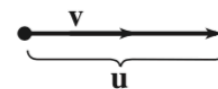
$\frac{\pi}{2} < \theta < \pi$
 $-1 < \cos \theta < 0$
 Obtuse Angle



$\theta = \frac{\pi}{2}$
 $\cos \theta = 0$
 90° Angle



$0 < \theta < \frac{\pi}{2}$
 $0 < \cos \theta < 1$
 Acute Angle



$\theta = 0$
 $\cos \theta = 1$
 Same Direction

Definition of Orthogonal Vectors

The vectors \mathbf{u} and \mathbf{v} are **orthogonal** if $\mathbf{u} \cdot \mathbf{v} = 0$.

Try as a Class

Determining Orthogonal Vectors

Are the vectors $\mathbf{u} = \langle 2, -3 \rangle$ and $\mathbf{v} = \langle 6, 4 \rangle$ orthogonal?

Solution

Solution

Find the dot product of the two vectors.

$$\mathbf{u} \cdot \mathbf{v} = \langle 2, -3 \rangle \cdot \langle 6, 4 \rangle = 2(6) + (-3)(4) = 0$$

Because the dot product is 0, the two vectors are orthogonal (see Figure 3.36).

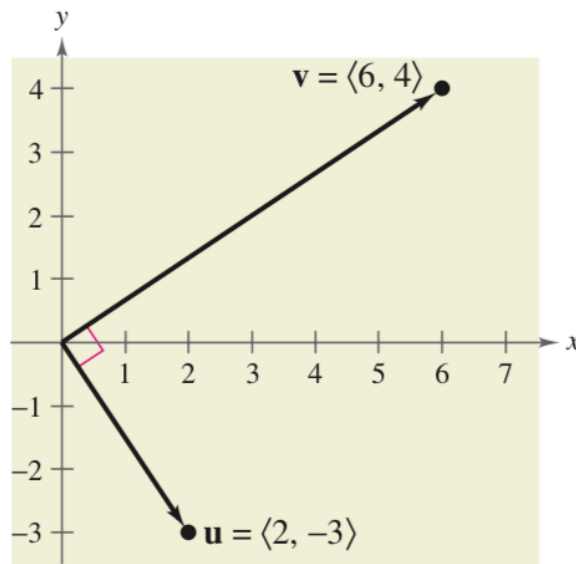


FIGURE 3.36

Definition of Vector Components

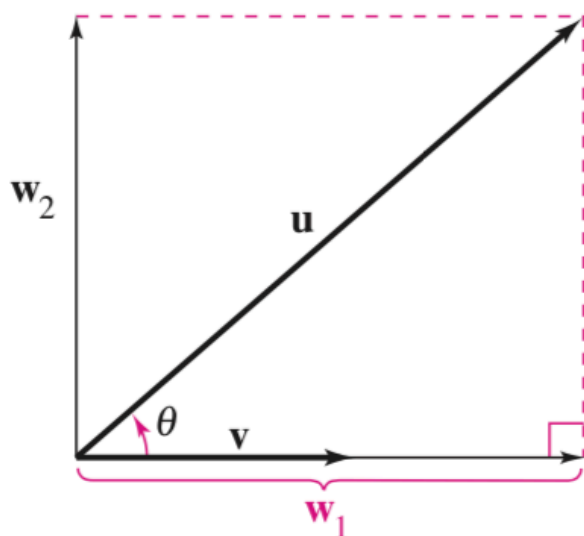
Let \mathbf{u} and \mathbf{v} be nonzero vectors such that

$$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2$$

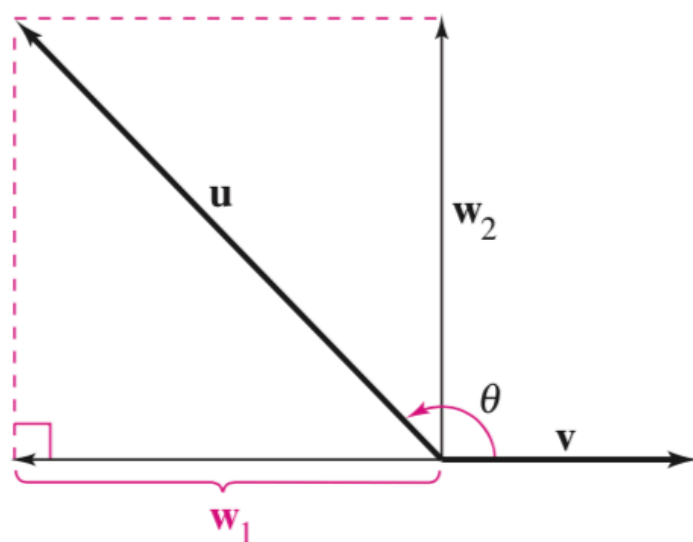
where \mathbf{w}_1 and \mathbf{w}_2 are orthogonal and \mathbf{w}_1 is parallel to (or a scalar multiple of) \mathbf{v} , as shown in Figure 3.38. The vectors \mathbf{w}_1 and \mathbf{w}_2 are called **vector components** of \mathbf{u} . The vector \mathbf{w}_1 is the **projection** of \mathbf{u} onto \mathbf{v} and is denoted by

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}} \mathbf{u}.$$

The vector \mathbf{w}_2 is given by $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1$.



θ is acute.



θ is obtuse.

FIGURE 3.38

Projection of \mathbf{u} onto \mathbf{v}

Let \mathbf{u} and \mathbf{v} be nonzero vectors. The projection of \mathbf{u} onto \mathbf{v} is

$$\text{proj}_{\mathbf{v}} \mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v}.$$

Try as a Class

Decomposing a Vector into Components

Find the projection of $\mathbf{u} = \langle 3, -5 \rangle$ onto $\mathbf{v} = \langle 6, 2 \rangle$. Then write \mathbf{u} as the sum of two orthogonal vectors, one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$.

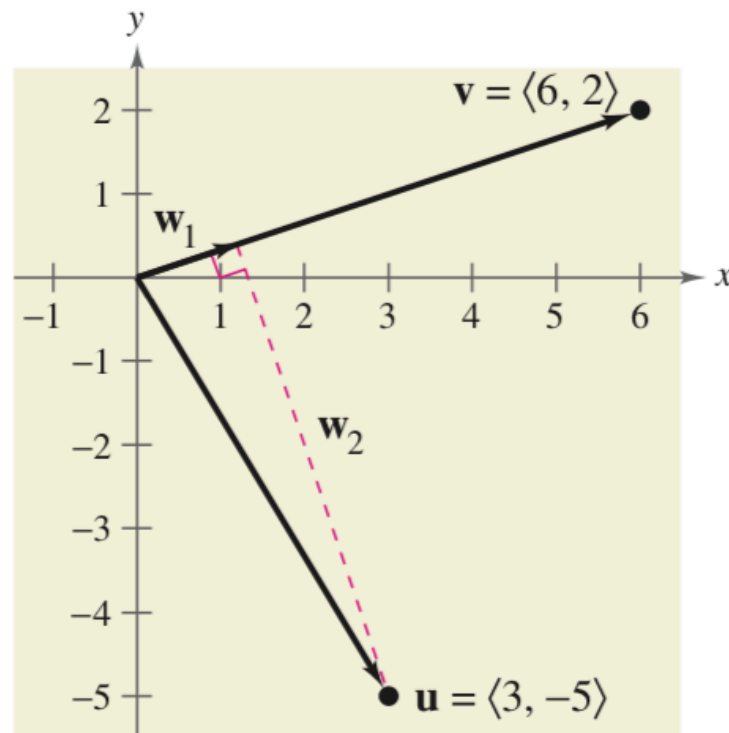


FIGURE 3.39

Solution

Solution

The projection of \mathbf{u} onto \mathbf{v} is

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}} \mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v} = \left(\frac{8}{40} \right) \langle 6, 2 \rangle = \left\langle \frac{6}{5}, \frac{2}{5} \right\rangle$$

as shown in Figure 3.39. The other component, \mathbf{w}_2 , is

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 3, -5 \rangle - \left\langle \frac{6}{5}, \frac{2}{5} \right\rangle = \left\langle \frac{9}{5}, -\frac{27}{5} \right\rangle.$$

So,

$$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2 = \left\langle \frac{6}{5}, \frac{2}{5} \right\rangle + \left\langle \frac{9}{5}, -\frac{27}{5} \right\rangle = \langle 3, -5 \rangle.$$

Try as a Class

Finding a Force

A 200-pound cart sits on a ramp inclined at 30° , as shown in Figure 3.40. What force is required to keep the cart from rolling down the ramp?

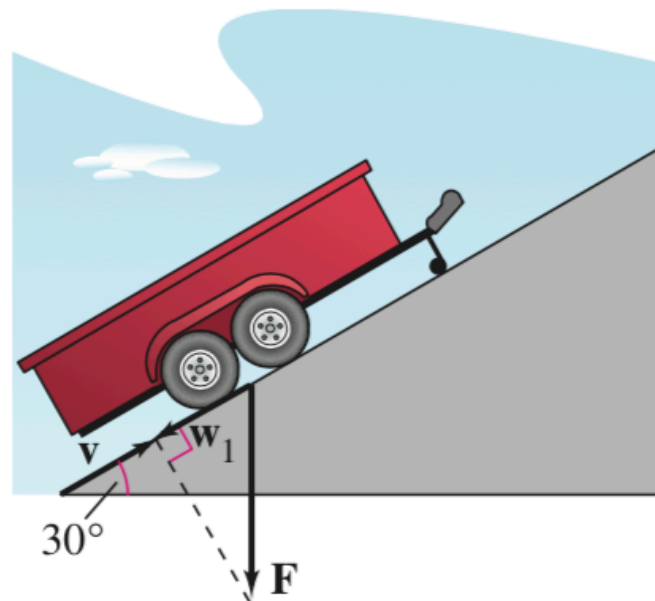


FIGURE 3.40

Solution

Solution

Because the force due to gravity is vertical and downward, you can represent the gravitational force by the vector

$$\mathbf{F} = -200\mathbf{j}. \quad \text{Force due to gravity}$$

To find the force required to keep the cart from rolling down the ramp, project \mathbf{F} onto a unit vector \mathbf{v} in the direction of the ramp, as follows.

$$\mathbf{v} = (\cos 30^\circ)\mathbf{i} + (\sin 30^\circ)\mathbf{j} = \frac{\sqrt{3}}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} \quad \text{Unit vector along ramp}$$

Therefore, the projection of \mathbf{F} onto \mathbf{v} is

$$\begin{aligned} \mathbf{w}_1 &= \text{proj}_{\mathbf{v}} \mathbf{F} \\ &= \left(\frac{\mathbf{F} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v} \\ &= (\mathbf{F} \cdot \mathbf{v}) \mathbf{v} \\ &= (-200) \left(\frac{1}{2} \right) \mathbf{v} \\ &= -100 \left(\frac{\sqrt{3}}{2} \mathbf{i} + \frac{1}{2} \mathbf{j} \right). \end{aligned}$$

The magnitude of this force is 100, and so a force of 100 pounds is required to keep the cart from rolling down the ramp.

Definition of Work

The **work** W done by a constant force \mathbf{F} as its point of application moves along the vector \overrightarrow{PQ} is given by either of the following.

1. $W = \|\text{proj}_{\overrightarrow{PQ}} \mathbf{F}\| \|\overrightarrow{PQ}\|$ Projection form
2. $W = \mathbf{F} \cdot \overrightarrow{PQ}$ Dot product form

Try as a Class

Finding Work

To close a sliding barn door, a person pulls on a rope with a constant force of 50 pounds at a constant angle of 60° , as shown in Figure 3.43. Find the work done in moving the barn door 12 feet to its closed position.

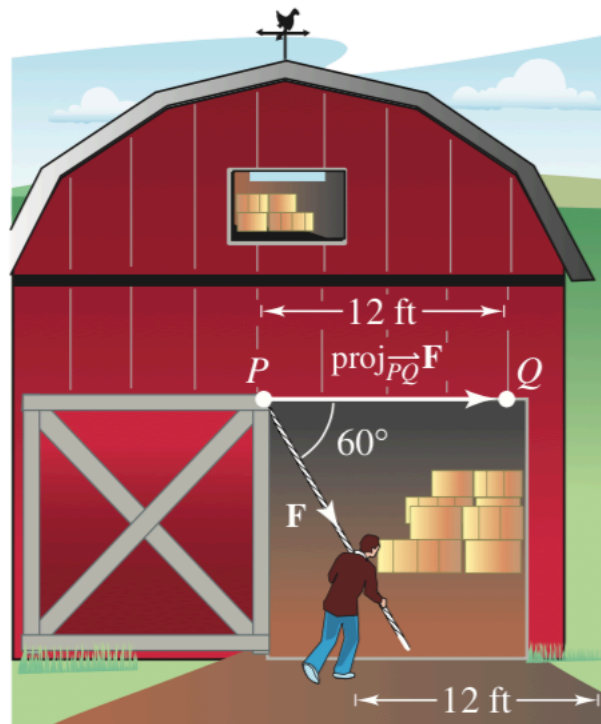


FIGURE 3.43

Solution

Solution

Using a projection, you can calculate the work as follows.

$$\begin{aligned} W &= \|\text{proj}_{\overrightarrow{PQ}} \mathbf{F}\| \|\overrightarrow{PQ}\| && \text{Projection form for work} \\ &= (\cos 60^\circ) \|\mathbf{F}\| \|\overrightarrow{PQ}\| \\ &= \frac{1}{2}(50)(12) = 300 \text{ foot-pounds} \end{aligned}$$

So, the work done is 300 foot-pounds. You can verify this result by finding the vectors \mathbf{F} and \overrightarrow{PQ} and calculating their dot product.