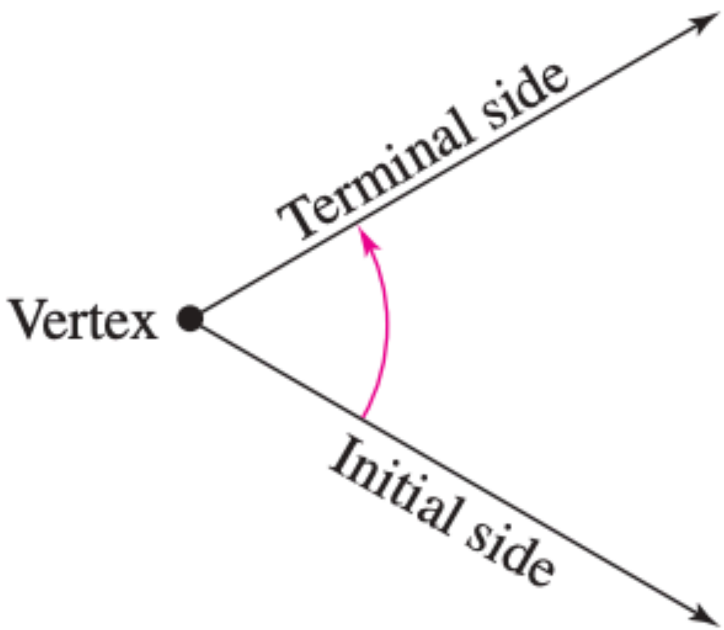
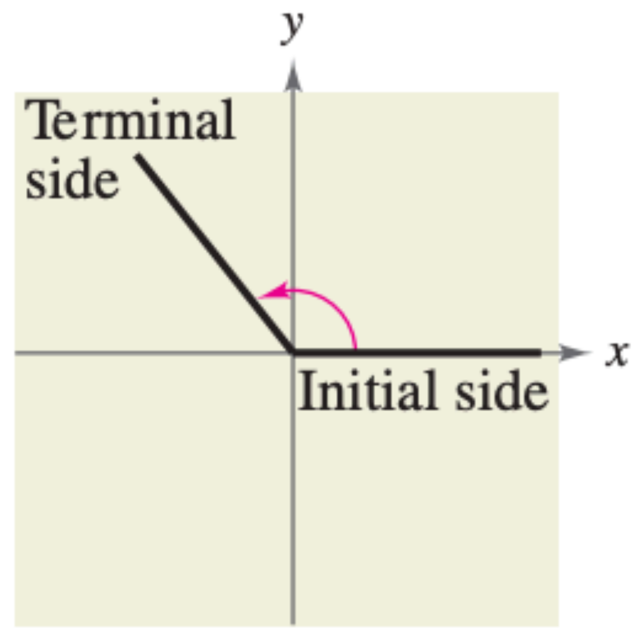
# Sec. 1.1

# How to measure angles…




Angle
FIGURE 1.1




Angle in standard position
FIGURE 1.2

# Radians!

**Definition of Radian**

One **radian** is the measure of a central angle $\theta$ that intercepts an arc $s$ equal in length to the radius $r$ of the circle. See Figure 1.5. Algebraically, this means that

$$\theta = \frac{s}{r}$$

where $\theta$ is measured in radians.

# Angles and the Cartesian Graph

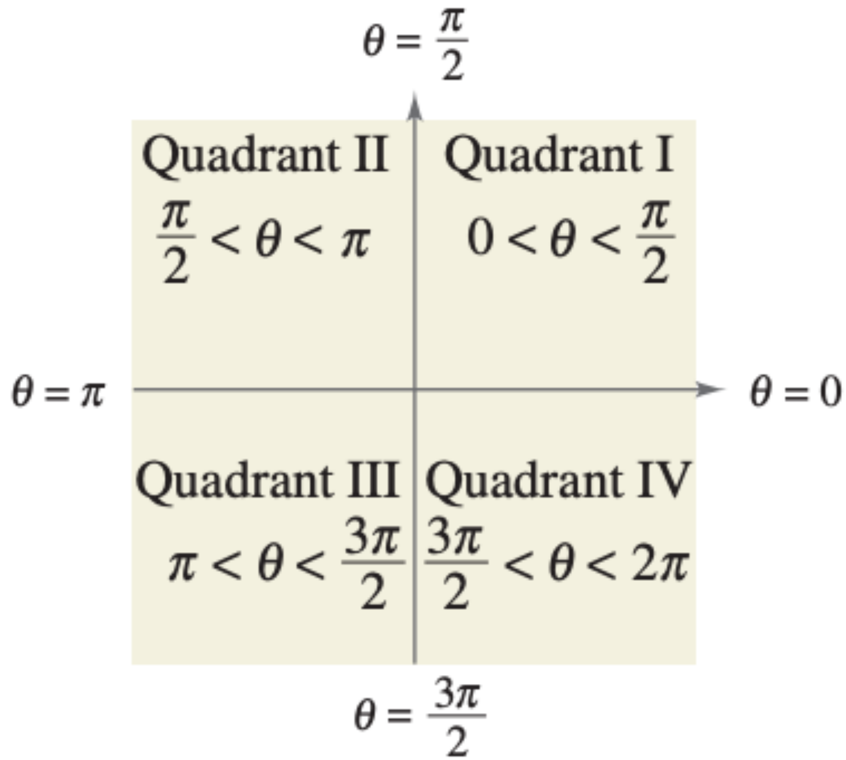



FIGURE 1.8

# Types of Angles


Complementary angles　　　　Supplementary angles

FIGURE 1.12

# But… What about degrees?

## Conversions Between Degrees and Radians

**1.** To convert degrees to radians, multiply degrees by $\dfrac{\pi \text{ rad}}{180°}$.

**2.** To convert radians to degrees, multiply radians by $\dfrac{180°}{\pi \text{ rad}}$.

To apply these two conversion rules, use the basic relationship $\pi \text{ rad} = 180°$. (See Figure 1.14.)

# Examples- Radians and Degrees

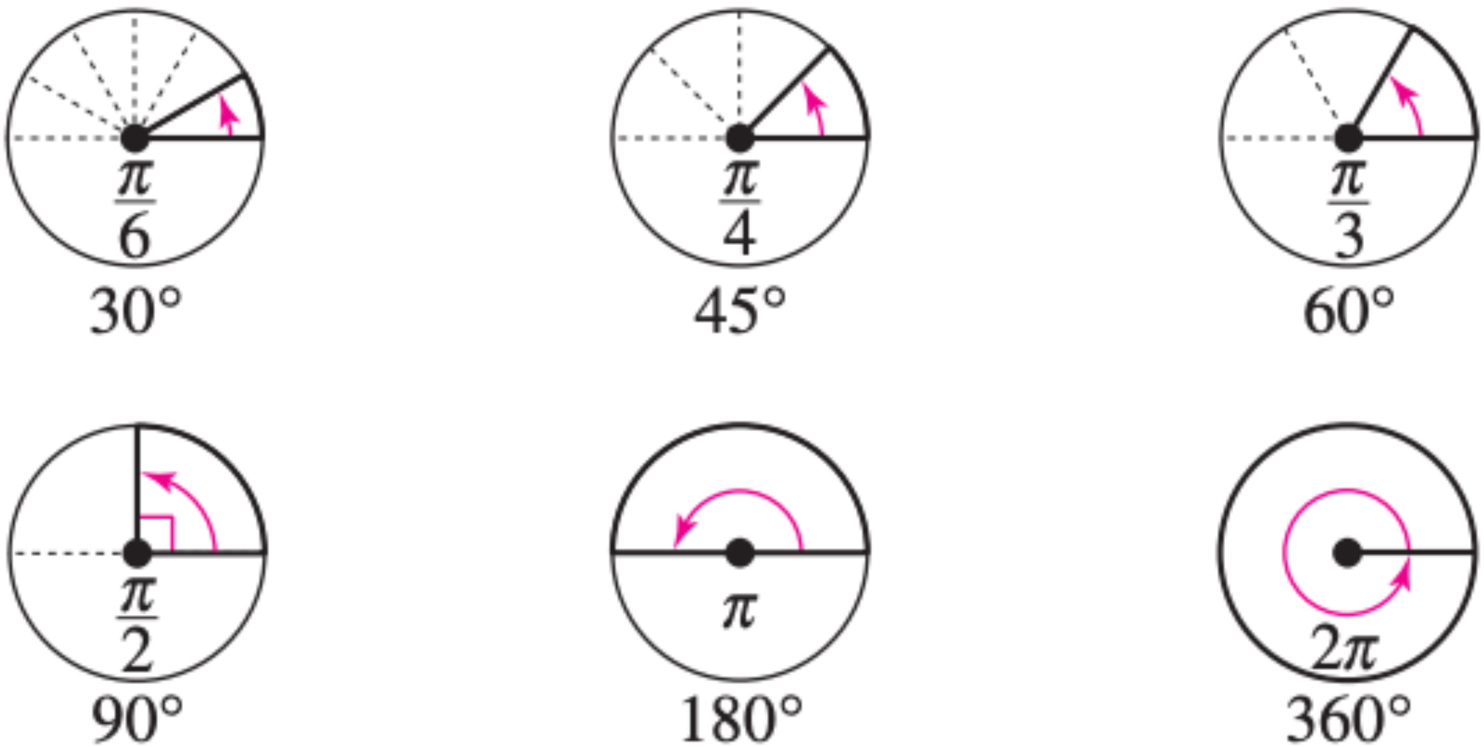



FIGURE 1.14

# …more examples

## Converting from Degrees to Radians

**a.** $135° = (135\ \cancel{\text{deg}})\left(\dfrac{\pi\ \text{rad}}{180\ \cancel{\text{deg}}}\right) = \dfrac{3\pi}{4}$ radians — Multiply by $\pi/180$.

**b.** $540° = (540\ \cancel{\text{deg}})\left(\dfrac{\pi\ \text{rad}}{180\ \cancel{\text{deg}}}\right) = 3\pi$ radians — Multiply by $\pi/180$.

**c.** $-270° = (-270\ \cancel{\text{deg}})\left(\dfrac{\pi\ \text{rad}}{180\ \cancel{\text{deg}}}\right) = -\dfrac{3\pi}{2}$ radians — Multiply by $\pi/180$.

**CHECK*Point*** Now try Exercise 57.

## Converting from Radians to Degrees

**a.** $-\dfrac{\pi}{2}\ \text{rad} = \left(-\dfrac{\pi}{2}\ \cancel{\text{rad}}\right)\left(\dfrac{180\ \text{deg}}{\pi\ \cancel{\text{rad}}}\right) = -90°$ — Multiply by $180/\pi$.

**b.** $\dfrac{9\pi}{2}\ \text{rad} = \left(\dfrac{9\pi}{2}\ \cancel{\text{rad}}\right)\left(\dfrac{180\ \text{deg}}{\pi\ \cancel{\text{rad}}}\right) = 810°$ — Multiply by $180/\pi$.

**c.** $2\ \text{rad} = (2\ \cancel{\text{rad}})\left(\dfrac{180\ \text{deg}}{\pi\ \cancel{\text{rad}}}\right) = \dfrac{360°}{\pi} \approx 114.59°$ — Multiply by $180/\pi$.

**CHECK*Point*** Now try Exercise 61.

# Coterminal Angle

**a.** For the positive angle $13\pi/6$, subtract $2\pi$ to obtain a coterminal angle

$$\frac{13\pi}{6} - 2\pi = \frac{\pi}{6}.$$

See Figure 1.9.

**b.** For the positive angle $3\pi/4$, subtract $2\pi$ to obtain a coterminal angle

$$\frac{3\pi}{4} - 2\pi = -\frac{5\pi}{4}.$$

See Figure 1.10.

**c.** For the negative angle $-2\pi/3$, add $2\pi$ to obtain a coterminal angle

$$-\frac{2\pi}{3} + 2\pi = \frac{4\pi}{3}.$$

See Figure 1.11.

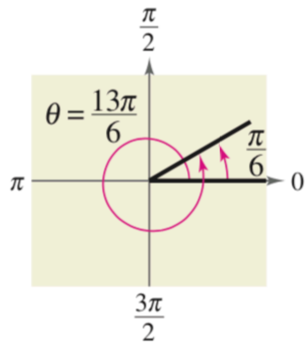



FIGURE 1.9

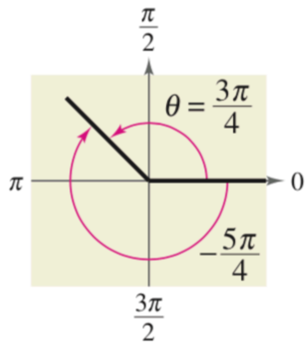



FIGURE 1.10

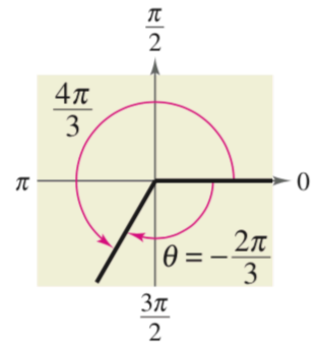



FIGURE 1.11

# Length of the exterior edge of a circle…

## Arc Length

For a circle of radius $r$, a central angle $\theta$ intercepts an arc of length $s$ given by

$$s = r\theta \qquad \text{Length of circular arc}$$

where $\theta$ is measured in radians. Note that if $r = 1$, then $s = \theta$, and the radian measure of $\theta$ equals the arc length.

# Example:

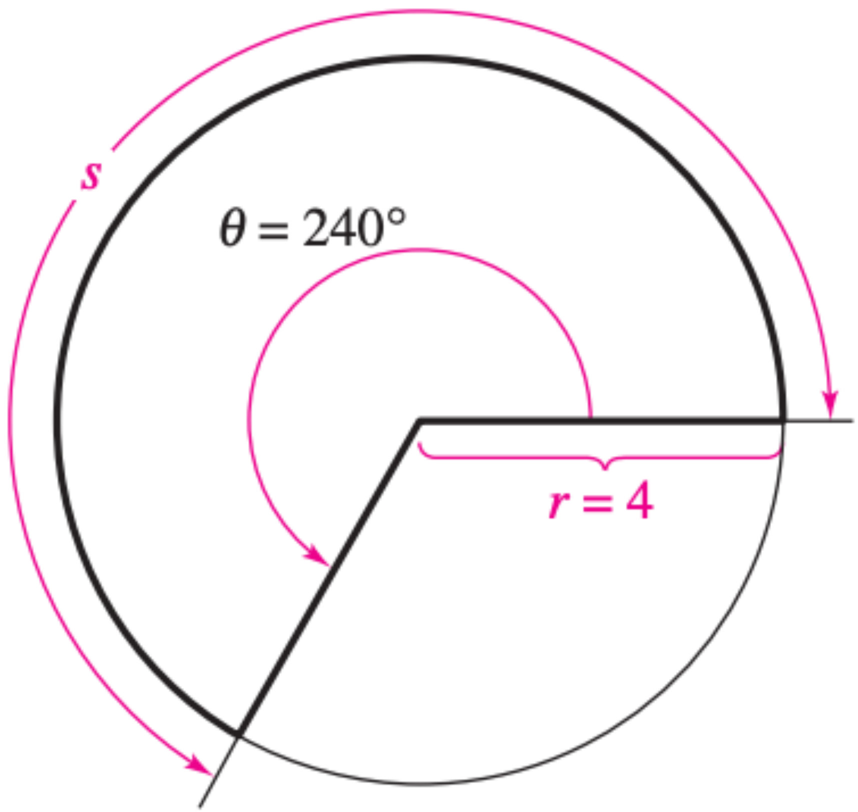



FIGURE 1.15

# So, we know the radius is 4. What's the circumference?

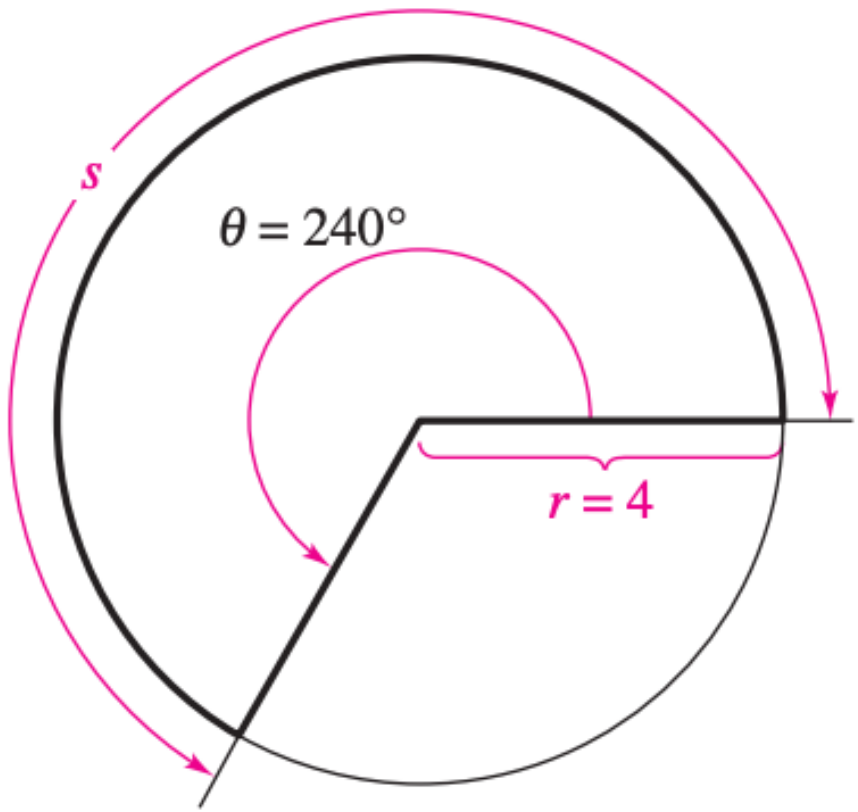



FIGURE 1.15

# Spinning fast = going fast?

## Linear and Angular Speeds

Consider a particle moving at a constant speed along a circular arc of radius $r$. If $s$ is the length of the arc traveled in time $t$, then the **linear speed** $v$ of the particle is

$$\text{Linear speed } v = \frac{\text{arc length}}{\text{time}} = \frac{s}{t}.$$

Moreover, if $\theta$ is the angle (in radian measure) corresponding to the arc length $s$, then the **angular speed** $\omega$ (the lowercase Greek letter omega) of the particle is

$$\text{Angular speed } \omega = \frac{\text{central angle}}{\text{time}} = \frac{\theta}{t}.$$

# Try as a class

The second hand of a clock is 10.2 centimeters long, as shown in Figure 1.16. Find the linear speed of the tip of this second hand as it passes around the clock face.


FIGURE 1.16

# Solution

## Solution

In one revolution, the arc length traveled is

$$\begin{aligned} s &= 2\pi r \\ &= 2\pi(10.2) && \text{Substitute for } r. \\ &= 20.4\pi \text{ centimeters.} \end{aligned}$$

The time required for the second hand to travel this distance is

$$t = 1 \text{ minute} = 60 \text{ seconds.}$$

So, the linear speed of the tip of the second hand is

$$\begin{aligned} \text{Linear speed} &= \frac{s}{t} \\ &= \frac{20.4\pi \text{ centimeters}}{60 \text{ seconds}} \\ &\approx 1.068 \text{ centimeters per second.} \end{aligned}$$

# Try as a Class

## Finding Angular and Linear Speeds

The blades of a wind turbine are 116 feet long (see Figure 1.17). The propeller rotates at 15 revolutions per minute.

**a.** Find the angular speed of the propeller in radians per minute.

**b.** Find the linear speed of the tips of the blades.


FIGURE 1.17

# Solution

## Solution

**a.** Because each revolution generates $2\pi$ radians, it follows that the propeller turns $(15)(2\pi) = 30\pi$ radians per minute. In other words, the angular speed is

$$\begin{aligned}\text{Angular speed} &= \frac{\theta}{t}\\ &= \frac{30\pi \text{ radians}}{1 \text{ minute}} = 30\pi \text{ radians per minute.}\end{aligned}$$

**b.** The linear speed is

$$\begin{aligned}\text{Linear speed} &= \frac{s}{t}\\ &= \frac{r\theta}{t}\\ &= \frac{(116)(30\pi) \text{ feet}}{1 \text{ minute}} \approx 10{,}933 \text{ feet per minute.}\end{aligned}$$

# But what about area…?

## Area of a Sector of a Circle

For a circle of radius $r$, the area $A$ of a sector of the circle with central angle $\theta$ is given by

$$A = \frac{1}{2}r^2\theta$$

where $\theta$ is measured in radians.

# Try as a Class

## Area of a Sector of a Circle

A sprinkler on a golf course fairway sprays water over a distance of 70 feet and rotates through an angle of 120° (see Figure 1.19). Find the area of the fairway watered by the sprinkler.

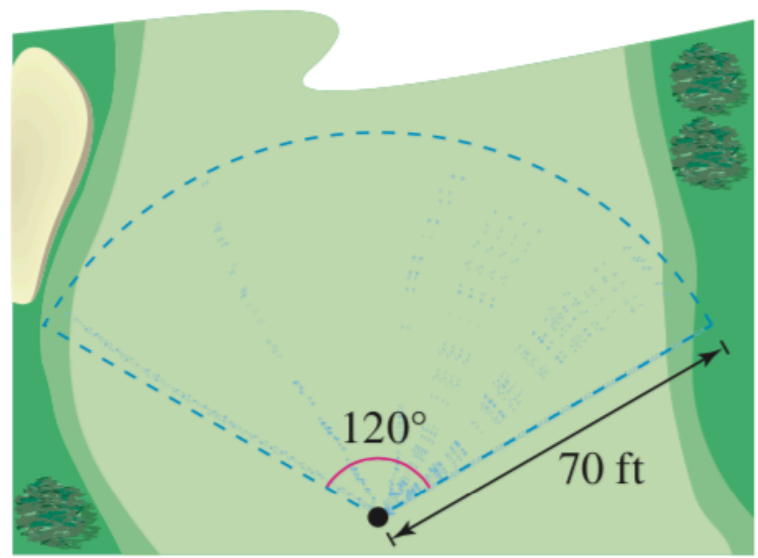



FIGURE 1.19

# Solution

## Solution

First convert 120° to radian measure as follows.

$$\theta = 120^\circ$$

$$= (120\ \cancel{\text{deg}})\left(\frac{\pi \text{ rad}}{180\ \cancel{\text{deg}}}\right) \qquad \text{Multiply by } \pi/180.$$

$$= \frac{2\pi}{3} \text{ radians}$$

Then, using $\theta = 2\pi/3$ and $r = 70$, the area is

$$A = \frac{1}{2}r^2\theta \qquad \text{Formula for the area of a sector of a circle}$$

$$= \frac{1}{2}(70)^2\left(\frac{2\pi}{3}\right) \qquad \text{Substitute for } r \text{ and } \theta.$$

$$= \frac{4900\pi}{3} \qquad \text{Simplify.}$$

$$\approx 5131 \text{ square feet.} \qquad \text{Simplify.}$$