Final Exam Study Guide

Name______________________________

1. Draw the unit circle and:

    (a) Label the angles: $\{-\frac{2\pi}{3}, 150°\}$

    (b) Label the following points: $\{\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)\}$

    (c) Label the point where: $\cos(\theta) = -\frac{\sqrt{2}}{2}$ and $\sin(\theta) = \frac{\sqrt{2}}{2}$

2. Rewrite the following complex numbers in terms of trig functions:

(a) $z = 13i + 4$ $\qquad$ $w = -5 + 3i$

(b) $z = 13i - 4i$ $\qquad$ $w = 3e^{i4\pi/3}$

3. Rewrite the following complex numbers in standard form:

(a) $z = 2\cos(60°) + 2i\sin(60°)$ $\qquad$ $w = 4(\cos(2\pi) + i\sin(2\pi))$

(b) $z = 9(\cos(\pi/2) + i\sin(\pi/2))$ $\qquad$ $w = \frac{1}{2}(\cos(210°) + i\sin(210°))$

4. Answer the following:

   (a) Tod needs to buy a new sprinkler. He finds a brand that he likes, but they only provide the area covered and radius. He knows the angle($\frac{\pi}{4}$) and radius($12m$) that the sprinkler needs to cover. What is the area the sprinkler needs to cover?

5. Answer the following:

   (a) A research group is studying a planet orbiting a distant star. They record the planet's position in it's orbit over several years. They discover the planet moves about $5°$ per week. If the planet is about $230,00,000km$ from the star. How far will it travel in 3 weeks?

   Hint: The problem is asking you to find the arc length of the path traveled by the planet after 3 weeks. To solve this problem, start by finding out how many degrees it moves in 3 weeks. It may help to draw a picture first.

6. Convert the angle from degrees to radians:

   (a) $17°$

7. Convert the angle from radians to degrees, and give its coterminal angle:

   (a) $\frac{13\pi}{4}$

8. A triangle has a **hypotenuse** of 13 and the length of the side **adjacent** to θ is 5 :

   (a) Draw the triangle. Find $\tan(\theta)$.

9. Show $\cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1$:

10. If $\cos(\theta) = \frac{\sqrt{3}}{2}$ and $\sin(\theta) = \frac{1}{2}$:

    (a) Draw the triangle, with hypotenuse 1 that this represents.
    Find $\tan(\theta)$.

11. Given $\frac{\cos(\frac{x}{5})}{2}$:

    (a) What is the amplitude? What is the period?

12. Sketch $\tan(x)$:

13. Graph the following trig. function:

(a) $y = \frac{x \sin(x)}{3}$

14. Give the exact solution, if possible, for the following inverse trigonometric functions exaluated at the point:

(a) $\arccos(9)$ $\arctan(1)$

(b) $\cos(x) = \frac{1}{\sqrt{3}}$ $\tan(x) = -1$

15. Factor the following:

(a) $\sec^3(x) - \sec(x)$

16. Use the fundamental identities to simplify the expression.

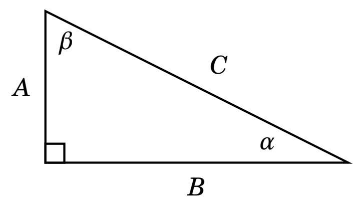
(a) $\frac{1}{1+\tan^2(x)}\sec^2(x)$

$1-(\cos(x)\tan(x))^2$

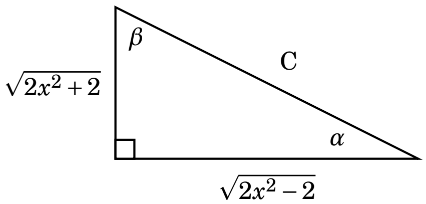
17. Given the following triangle and the following solutions:

$C \sin(\alpha) = 5$
$C \cos(\alpha) = 3$

find A, B, C, α, β.



18. Given the following triangle, find C and $sin(\alpha)$.



19. **Solve for x.**

Hint: If you apply the subsitution $u = \sin(x)$ *what would the quadratic formula tell you?*

Remember, the quadratic formula is $u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ *and solves* $au^2 + bu + c = 0$

YOU DO NOT NEED TO USE AN IDENTITY!!!!

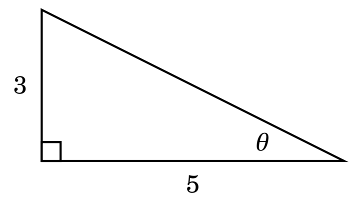
(a) $\sin^2(x) - 5\sin(x) = 6$

20. Prove the following:
    Hint: $\tan\left(\frac{\pi}{2}\right)$ is undefined. So we can't use one of the identites.
    (a) $\tan\left(\frac{\pi}{2}-u\right)=\cot(u)$

21. Rewrite the following terms using the Sum and Difference or Double-Angle Formulas:
    (a) $\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{5\pi}{12}\right)+\sin\left(\frac{5\pi}{12}\right)\cos\left(\frac{\pi}{12}\right)$
    $2\sin(15^\circ)\cos(15^\circ)$

22. Solve for u:
    Hint: $\tan\left(\frac{u}{2}\right)=\frac{\sin(u)}{1+\cos(u)}$
    (a) $\tan\left(\frac{u}{2}\right)(1+\cos(u))=\frac{\sqrt{3}}{2}$

23. Given the following triangle find: $\cos(2\theta)$
Hint: you do not have to find θ



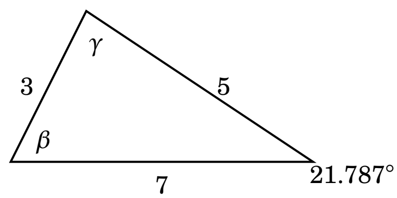
24. Solve using the product-to-sum formula:

(a) $\sin(7\theta)\sin(2\theta)$

$4\cos(2\theta)\cos(4\theta)$

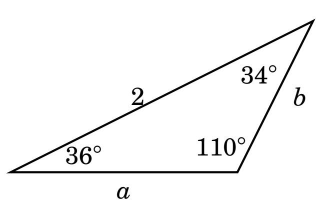
25. Given the following triangle find:

β, γ, and the area of the triangle



26. Given the following triangle find:

a, b, and the area of the triangle



27. Solve the problems below given the following two vectors:

$$\mathbf{u} = \langle 1,\ -3 \rangle \qquad\qquad \mathbf{v} = \langle -5,\ 4 \rangle$$

(a) $\mathbf{u} - 2\mathbf{v}$

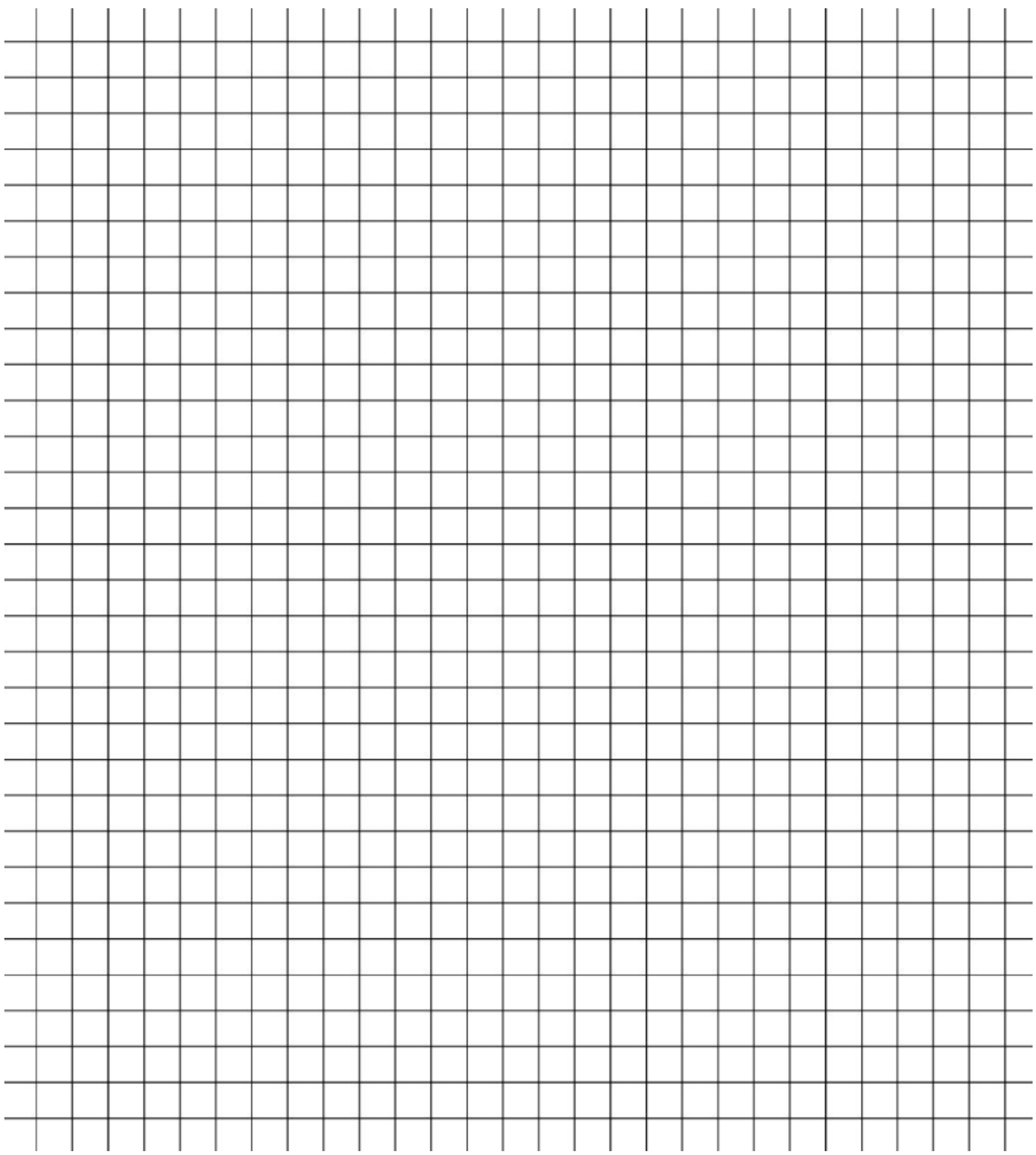
28. Find unit vectors in the direction of the provided vectors:

(a) $\mathbf{u} = \langle 5,\ 12 \rangle$

29. Find, and graph, the projection of $\mathbf{u}$ onto $\mathbf{v}$ ($\mathbf{w}_1 = \text{proj}_{\mathbf{v}}(\mathbf{u})$):

$\mathbf{u} = \langle 1,\ 3 \rangle$ $\qquad\qquad$ $\mathbf{v} = \langle 6,\ -5 \rangle$

(a)



30. Solve the problems below given the following two vectors:

$$\mathbf{u} = \langle 5,\ 1 \rangle \qquad\qquad \mathbf{v} = \langle 3,\ 2 \rangle$$

(a) Find the magnitude $\mathbf{u}$ and $\mathbf{v}$

(b) Find the angle between $\mathbf{u}$ and $\mathbf{v}$

(c) Find the component of $\mathbf{u}$ orthognal/normal to $\mathbf{v}$

31. Simplify the following:

(a)
$$(1+i)(12+i12)$$

(b)
$$\frac{3-i}{3}-\frac{i4}{1-4i}$$

32. Solve the problems below given the following two vectors:

$$\mathbf{u} = \langle 2,\ 3\rangle \qquad\qquad \mathbf{v} = \langle -9,\ 6\rangle$$

(a) $\mathbf{u} - 2\mathbf{v}$

$\mathbf{u} \cdot \mathbf{v}$

33. Find unit vectors in the direction of the provided vectors:

(a) $\mathbf{u} = \langle 4,\ 3\rangle$

$\mathbf{v} = \langle 2,\ -9\rangle$

34. Write the following vectors as a linear combination of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$ then graph $2\mathbf{u} - \mathbf{v}$:

$\mathbf{u} = \langle 4,\ 1 \rangle$ $\mathbf{v} = \langle -3,\ 2 \rangle$

(a)

35. Find, and graph, the projection of $\mathbf{u}$ onto $\mathbf{v}$ ($\mathbf{w}_1 = \text{proj}_{\mathbf{v}}(\mathbf{u})$):

$\mathbf{u} = \langle 2,\ 3 \rangle$ $\mathbf{v} = \langle 3,\ -5 \rangle$

(a)

36. Solve the problems below given the following two vectors:

$$\mathbf{u} = \langle 4,\ 1 \rangle \qquad\qquad \mathbf{v} = \langle -3,\ 2 \rangle$$

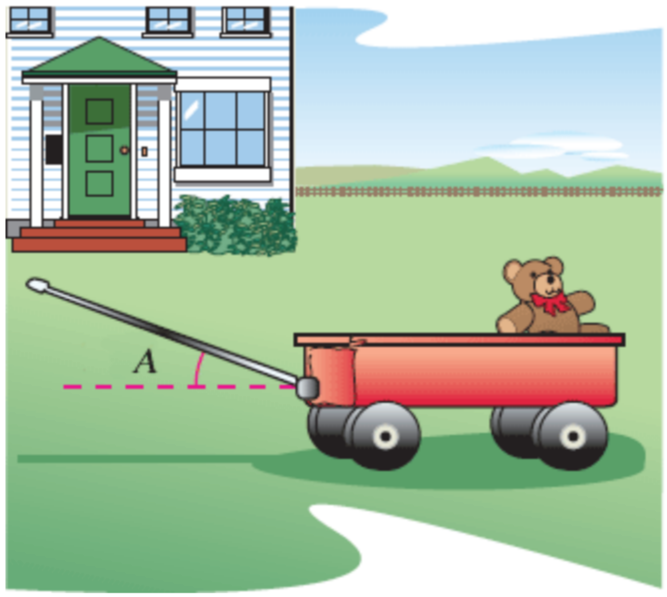
(a) Find the angle between **u** and **v**

(b) Find the magnitude **u** and **v**

(c) Find the component of **u** orthognal/normal to **v**

37. A child pulls a toy wagon by exerting a constant force of 27 pounds on a handle that makes a 34deg angle with the horizontal (see figure). Determine the work done in pulling the wagon 62 feet.

(a)



38. Find two vectors, in opposite directions, that are orthognal/normal to $\mathbf{u} = \langle -2,\ -3 \rangle$:

(a)

39. Simplify the following:

(a)
$\frac{2}{4-i}$

$(12+i)-(1+i12)$

(b)
$\frac{4-i}{5} - \frac{i}{3-4i}$

$(4i)(3+5i)$

40. Find each function given the constant a and the function's zeros/roots and their multiplicity:

(a) $f(x)$ has the zeros-
2 with a multiplitiKcy of 1
-3 with a multiplity of 3,
and $a = 2$

41. Solve the following:

(a) $\sqrt[4]{81\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)}$

(b) $(1 - i4)^3$

42. Graph the following two trig. functions:

(a) $y = \tan\left(\frac{x}{5}\right)$ $\qquad$ $y = 2x\cos(x)$

43. Give the exact solution, if possible, for the following inverse trigonometric functions exaluated at the point:

(a) $\arctan(0)$ $\qquad$ $\arcsin(9)$

(b) $\cos(x) = \frac{1}{2}$ $\qquad$ $\frac{1}{2}\tan(x) = \frac{1}{2}$

(c) $\arccos(\cos(30°))$ $\qquad$ $\tan\left(\arccos\left(\frac{\sqrt{2}}{2}\right)\right)$

44. What is the period of the following trigonometric functions:

(a) $\csc(3x)$ $\qquad$ $2\cos\left(\frac{x}{5}\right)$

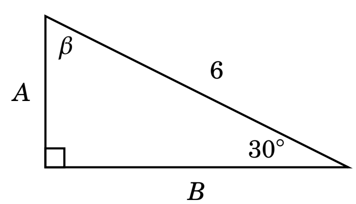
45. What is the domain of:
Hint: I'm asking what values can x be? Your answer should be an inequality or interval.

(a) $\sec(x)$ $\quad$ $\csc(x)$

46. Factor each of the following:

(a) $\sec^3(x) + \sec(x)$ $\quad$ $\cos^2(x)\tan^2(x) - \cot^2(x)\sin^2(x)$

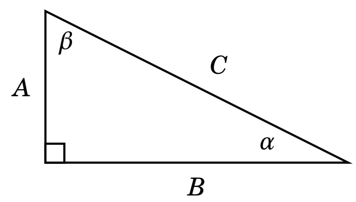
47. Given the following triangle find A, B, β:



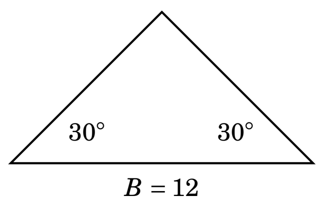
48. Given the following triangle and the following solutions:

$C\sin(\alpha) = 5$
$C\sin(\beta) = 3$

find A, B, C, α, β.



49. Given the following triangle find the height of the triangle:



50. The sun is 35° above the horizon. Find the length of a shadow cast by a building that is 550 feet tall and draw the corresponding triangle:

51. Use the given conditions to find the values of all six trigonometric functions.
$\cos(x) = -\frac{3}{7}$, $sin(x) < 0$

52. Use the fundamental identities to simplify the expression.

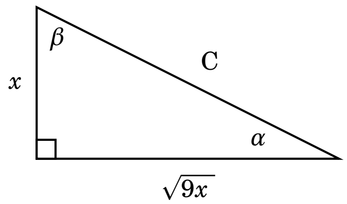
(a) $\frac{\sec(x)\tan(x)}{cot(x)}$

$2\cos(x) - \frac{2}{sec^3(x)}$

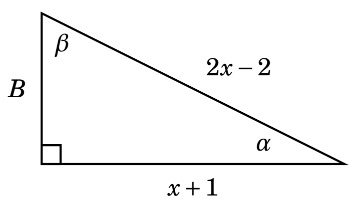
(b) $\frac{sec(x)}{csc(x)} cot^2(x) - cot(x)$

$\frac{csc(x)}{sec(x)} tan(x) + tan^2(x)$

53. Given the following triangle, find C and give β using one of the arc-trig functions.



54. Given the following triangle, find B and give α using one of the arc-trig functions.



55. Draw the unit circle and:

(a) Label the angles: $\{90°, \frac{2\pi}{3}, 150°, \frac{3\pi}{2}\}$

(b) Label the following points: $\{\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)\}$

56. Give the supplementary angle for:

(a) $\frac{3\pi}{5}$ $\frac{\pi}{8}$

57. Give the complementary angle for:

(a) $\frac{\pi}{5}$ $\frac{3\pi}{8}$

58. Find the angle in radians provided:

(a) $r = 10$, Area: 50π $r = 3$, Arc length: $\frac{6\pi}{5}$

59. Find the area:

(a) $r = \frac{1}{2}$, Angle: 3π $r = 2$, Angle: $\frac{5\pi}{6}$

60. Find the arc length:

(a) $r = 5$, Angle: $\frac{\pi}{12}$ $r = 2$, Angle: $\frac{\pi}{3}$

61. What quadrant is the following angle in:

(a) $405°$ $\frac{3\pi}{2}$

62. Convert the angle from degrees to radians:

(a) $10°$ $175°$

63. Convert the angle from radians to degrees:

(a) $\frac{2\pi}{5}$ $\frac{7\pi}{18}$

64. If $\sec(t) = \frac{2}{\sqrt{3}}$ find:

(a) $\cos(t)$

$\sin(t)$

65. Find the reference angle:

(a) $320°$

$\frac{5\pi}{3}$

66. A triangle has a hypotenuse of 13 and the length of the side opposite θ is 5 :

(a) Draw the triangle.

Find $\tan(\theta)$.

67. Show $\cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1$:

68. If $\cos(\theta) = \frac{3}{5}$ and $\sin(\theta) = \frac{4}{5}$:

(a) Draw the triangle.

Find $\cot(\theta)$.

69. Sketch $3\sin\left(\frac{x}{2}\right)$:

(a) What is the amplitude?

What is the period?

70. Sketch $3\csc(x)$:

71. Sketch $-\tan\left(\frac{x}{2}\right)$: