

Study Guide (Math 111: College Algebra)

This study guide is meant to assist students in preparing for the departmental college algebra final. It is not meant to be all-inclusive and is intended to help supplement a student's review as they study their previous exams and course materials. These problems are not exactly like the college algebra final problems and there may be problems on the final that are not included in the list below.

1. Factor $x^4 - 2x^3 - 8x^2$ completely

Solution:

$$\begin{aligned}x^4 - 2x^3 - 8x^2 &= x^2(x^2 - 2x - 8) \\ &= x^2(x - 4)(x + 2)\end{aligned}$$

2. Factor out the greatest common term: $150x^3yz^7 - 32xy^3z^{\frac{5}{2}}$

Solution:

$$150x^3yz^7 - 32xy^3z^{\frac{5}{2}} = 2xyz^{\frac{5}{2}}(75x^2z^{\frac{9}{2}} - 16y^2)$$

3. Solve: $12x^2 + 23x = -10$

Solution:

$$\begin{aligned}12x^2 + 23x &= -10 \\ 12x^2 + 23x + 10 &= 0 \\ 12x^2 + 15x + 8x + 10 &= 0 \\ 3x(4x + 5) + 2(4x + 5) &= 0 \\ (3x + 2)(4x + 5) &= 0 \\ 3x + 2 = 0 \quad 4x + 5 &= 0 \\ x &= \frac{-2}{3}, \frac{-5}{4}\end{aligned}$$

4. Solve: $x^4 - 5x^2 + 4 = 0$

Solution:

$$\begin{aligned}x^4 - 5x^2 + 4 &= 0 \\(x^2)^2 - 5x^2 + 4 &= 0 \\(x^2 - 4)(x^2 - 1) &= 0 \\x^2 - 4 &= 0 \\x^2 &= 4 \\x &= \pm 2 \\x^2 - 1 &= 0 \\x^2 &= 1 \\x &= \pm 1 \\x &= 1, -1, 2, -2\end{aligned}$$

5. Solve: $\frac{x-8}{3} + \frac{x-3}{2} = 0$

Solution:

$$\begin{aligned}\frac{x-8}{3} + \frac{x-3}{2} &= 0 \\\frac{2(x-8)}{6} + \frac{3(x-3)}{6} &= 0 \\\frac{2x-16+3x-9}{6} &= 0 \\\frac{5x-25}{6} &= 0 \\5x-25 &= 0 \\x &= 5\end{aligned}$$

6. Solve: $9 + \sqrt{x-3} = 12$

Solution:

$$9 + \sqrt{x - 3} = 12$$

$$\sqrt{x - 3} = 3$$

$$x - 3 = 9$$

$$x = 12$$

Check: $9 + \sqrt{12 - 3} = 9 + 3 = 12$

7. Solve: $|x - 3| + 2 = 7$

Solution:

$$|x - 3| + 2 = 7$$

$$|x - 3| = 5$$

$$x - 3 = 5 \text{ or } x - 3 = -5$$

$$x = 8, -2$$

8. Solve: $x^2 - 25 < 0$

Solution: Set =

$$x^2 - 25 = 0$$

$$x^2 = 25$$

$$x = \pm 5$$

	interval	test pt	sign for f(test point)
Set up number line and test points	$(-\infty, -5)$	-10	+
	$(-5, 5)$	0	-
	$(5, \infty)$	10	+

Select intervals with < 0 :

$$(-5, 5)$$

9. Solve: $2|x - 4| > 8$

Solution:

$$\begin{aligned}2|x - 4| &> 8 \\|x - 4| &> 4 \\x - 4 &> 4 \text{ or } x - 4 < -4 \\x &> 8 \text{ or } x < 0\end{aligned}$$

10. Solve: $|4x + 7| \leq 9$

Solution:

$$\begin{aligned}|4x + 7| &\leq 9 \\-9 &\leq 4x + 7 \leq 9 \\-16 &\leq 4x \leq 2 \\-4 &\leq x \leq \frac{1}{2}\end{aligned}$$

11. Solve: $\log(x) + \log(x + 3) = 1$

Solution:

$$\begin{aligned}\log(x) + \log(x + 3) &= 1 \\\log(x(x + 3)) &= 1 \\\log_{10}(x(x + 3)) &= 1 \\10^1 &= x^2 + 3x \\0 &= x^2 + 3x - 10 \\0 &= (x + 5)(x - 2) \\x &= -5, 2\end{aligned}$$

Check: Notice that -5 does not work as it makes the inside of the logarithm negative.

Then $x = 2$

12. Solve: $e^{x+1} = 9$

Solution:

$$\begin{aligned}e^{x+1} &= 9 \\ \log_e 9 &= x + 1 \\ \ln 9 - 1 &= x\end{aligned}$$

13. Solve: $2^{3x-1} = \frac{1}{8}$

Solution:

$$\begin{aligned}2^{3x-1} &= \frac{1}{8} \\ 2^{3x-1} &= 2^{-3} \\ 3x - 1 &= -3 \\ 3x &= -2 \\ x &= \frac{-2}{3}\end{aligned}$$

14. Solve: $e^{2x} - e^x - 2 = 0$

Solution:

$$\begin{aligned}e^{2x} - e^x - 2 &= 0 \\(e^x)^2 - e^x - 2 &= 0 \\(e^x - 2)(e^x + 1) &= 0 \\e^x - 2 = 0 \text{ or } e^x + 1 &= 0 \\e^x &= 2 \\\ln 2 &= x \\e^x = -1 &\text{ not possible}\end{aligned}$$

15. Solve: $\ln(4x + 6) - \ln(x + 5) = \ln(x)$

Solution:

$$\begin{aligned}\ln(4x + 6) - \ln(x + 5) &= \ln(x) \\\ln\left(\frac{4x + 6}{x + 5}\right) &= \ln(x) \\\frac{4x + 6}{x + 5} &= x \\4x + 6 &= x(x + 5) \\4x + 6 &= x^2 + 5x \\0 &= x^2 + x - 6 \\0 &= (x + 3)(x - 2) \\x &= -3, 2\end{aligned}$$

Notice that -3 doesn't work, so $x = 2$

16. Solve the system of equations: $\begin{cases} x + y = 9 \\ 2x - 3y = -2 \end{cases}$

Solution:

$$-2(x + y) = -2 * 9$$

$$2x - 3y = -2$$

$$-2x - 2y = -18$$

$$2x - 3y = -2$$

$$-5y = -20$$

$$y = 4$$

Then as $x + y = 9$, $x = 5$
(5, 4)

17. Solve the system of equations: $\begin{cases} 0.2x - 0.3y = 0.3 \\ 0.4x + 0.6y = -0.2 \end{cases}$

Solution:

$$10(0.2x - 0.3y) = 10 * 0.3$$

$$10(0.4x + 0.6y) = 10 * -0.2$$

$$2x - 3y = 3$$

$$4x + 6y = -2$$

$$2 * (2x - 3y) = 2 * 3$$

$$4x + 6y = -2$$

$$4x - 6y = 6$$

$$4x + 6y = -2$$

$$8x = 4$$

$$x = \frac{1}{2}$$

Then as $2x - 3y = 3$:
 $1 - 3y = 3$

$$\begin{aligned} -3y &= 2 \\ \text{or } y &= \frac{-2}{3} \\ \left(\frac{1}{2}, \frac{-2}{3}\right) \end{aligned}$$

18. Express as the sum/difference of logarithms: $\log_2 \left(\frac{4x^2\sqrt{y}}{z^3} \right)$

Solution:

$$\begin{aligned} \log_2 \left(\frac{4x^2\sqrt{y}}{z^3} \right) &= \log_2 4x^2\sqrt{y} - \log_2 z^3 \\ &= \log_2 4 + \log_2 x^2 + \log_2 \sqrt{y} - \log_2 z^3 \\ &= 2 + 2\log_2 x + \frac{1}{2}\log_2 y - 3\log_2 z \end{aligned}$$

19. Express as a single logarithm: $\log 5 + 2\log 2 + \log 3 - \log 6$

Solution:

$$\begin{aligned} \log 5 + 2\log 2 + \log 3 - \log 6 &= \log 5 + \log 4 + \log 3 - \log 6 \\ &= \log 60 - \log 6 \\ &= \log \left(\frac{60}{6} \right) \\ &= \log 10 \\ &= 1 \end{aligned}$$

20. Find all real and complex zeros of the polynomial, given that 2 is a zero:

$$f(x) = x^4 - 7x^3 + 12x^2 + 4x - 16$$

$$\begin{array}{r|rrrrr} \text{Solution: } 2 & 1 & -7 & 12 & 4 & -16 \\ & & 2 & -10 & 4 & 16 \\ \hline & 1 & -5 & 2 & 8 & 0 \end{array}$$

Then $f(x) = (x - 2)(x^3 - 5x^2 + 2x + 8)$

Possible zeros: $\pm 8, \pm 4, \pm 2, \pm 1$

$$\begin{array}{r|rrrr} 2 & 1 & -5 & 2 & 8 \\ & & 2 & -6 & -8 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

Then $f(x) = (x - 2)(x - 2)(x^2 - 3x - 4) = (x - 2)^2(x - 4)(x + 1)$

Zeros: $x = 2, 4, -1$

21. Find all real and complex zeros of the polynomial: $f(x) = x^5 - x$

Solution:

$$\begin{aligned} x^5 - x &= x(x^4 - 1) \\ &= x(x^2 - 1)(x^2 + 1) \\ &= x(x - 1)(x + 1)(x - i)(x + i) \end{aligned}$$

Zeros: $x = 0, 1, -1, i, -i$

22. Perform the division:

$$(4x^3 - 3x^2 + x + 7) \div (x - 2)$$

Solution:

Using Synthetic

$$\begin{array}{r|rrrr}
 & 4 & -3 & 1 & 7 \\
 2 & & 8 & 10 & 22 \\
 \hline
 & 4 & 5 & 11 & 29
 \end{array}$$

and $(4x^2 + 5x + 11) + \frac{29}{x-2}$

Using Long

$$\begin{array}{r}
 4x^2 + 5x + 11 \overline{) 4x^3 - 3x^2 + x + 7} \\
 \underline{4x^3 - 8x^2} \\
 5x^2 + x + 7 \\
 \underline{5x^2 + 10x} \\
 11x + 7 \\
 \underline{11x + 22} \\
 29
 \end{array}$$

$(4x^2 + 5x + 11) + \frac{29}{x-2}$

23. Assume the time (t) it takes to frame a house varies inversely as the number (N) of people who work on the site. If it takes 72 hours for 9 people to frame a house, how long will it take 12 people to complete the same job?

Solution:

$$\begin{aligned}
 t &= \frac{k}{n} \\
 72 &= \frac{k}{9} \\
 72 * 9 &= k \\
 648 &= k
 \end{aligned}$$

Then, to find n:

$$\begin{aligned}
 t &= \frac{648}{n} \\
 &= \frac{648}{12} \\
 &= 54
 \end{aligned}$$

24. Simplify: $\sqrt{81x^9y^4}$

Solution:

$$\begin{aligned}\sqrt{81} * \sqrt{x^9} * \sqrt{y^4} &= 9\sqrt{x^8 * x} * y^2 \\ &= 9x^4 y^2 \sqrt{x}\end{aligned}$$

25. Express in the form of $a + bi$

$$(3 - 2i)(9 + \sqrt{-25})$$

Solution:

$$\begin{aligned}(3 - 2i)(9 + \sqrt{-25}) &= (3 - 2i)(9 + 5i) \\ &= 27 - 18i + 15i - 10i^2 \\ &= 27 - 3i + 10 \\ &= 37 - 3i\end{aligned}$$

26. Perform the subtraction:

$$\frac{x}{x^2 - 1} - \frac{3}{x^2 + 4x - 5}$$

Solution:

$$\begin{aligned}\frac{x}{x^2 - 1} - \frac{3}{x^2 + 4x - 5} &= \frac{x}{(x - 1)(x + 1)} - \frac{3}{(x + 5)(x - 1)} \\ &= \frac{x(x + 5)}{(x - 1)(x + 1)(x + 5)} - \frac{3(x + 1)}{(x + 5)(x - 1)(x + 1)} \\ &= \frac{x^2 + 5x - 3x - 3}{(x - 1)(x + 1)(x + 5)} \\ &= \frac{x^2 + 2x - 3}{(x - 1)(x + 1)(x + 5)} \\ &= \frac{(x + 3)(x - 1)}{(x - 1)(x + 1)(x + 5)} \\ &= \frac{x + 3}{(x + 1)(x + 5)}\end{aligned}$$

27. Find the equation of the line through the points (2, 1) and (-3, 0)

Solution: $m = \frac{1-0}{2-(-3)} = \frac{1}{5}$

Then,

$$y = mx + b$$

$$0 = \frac{1}{5} * -3 + b$$

$$0 = \frac{-3}{5} + b$$

$$b = \frac{3}{5}$$

So, $y = \frac{1}{5}x + \frac{3}{5}$

28. Find the x and y-intercepts of $f(x) = x^3 - 3x^2 - 4x + 12$

Solution:

x-intercepts: set $y = 0$

$$\begin{aligned} 0 &= x^3 - 3x^2 - 4x + 12 \\ &= x^2(x - 3) - 4(x - 3) \\ &= (x^2 - 4)(x - 3) \\ &= (x - 2)(x + 2)(x - 3) \end{aligned}$$

x-intercepts: (2, 0), (3, 0), (-2, 0)

y-intercepts: Set $x = 0$
(0, 12)

29. Find a_5 given $a_n = \frac{n^2 - 1}{n^2 + 1}$

$$\textbf{Solution: } a_5 = \frac{25-1}{25+1} = \frac{24}{26} = \frac{12}{13}$$

30. Evaluate the sum $\sum_{k=1}^3 k^2 + k - 1$

Solution:

$$a_1 = 1 + 1 - 1 = 1$$

$$a_2 = 4 + 2 - 1 = 5$$

$$a_3 = 9 + 3 - 1 = 11$$

$$\sum_{k=1}^3 k^2 + k - 1 = 17$$

31. Find the domain of each of the following:

(a) $f(x) = \frac{x}{x^2 - 9}$

Solution: $x^2 - 9 = 0$
Then, $\{x|x \neq \pm 3\}$

(b) $g(x) = \sqrt{9-x}$

Solution: $9-x \geq 0$
Then, $\geq x$ or $\{x|x \leq 9\}$

(c) $h(x) = \ln(3x-7)$

Solution: $3x-7 > 0$
 $3x > 7$
 $x > \frac{7}{3}$

$$\{x|x > \frac{7}{3}\}$$

32. For $f(x) = 2x - 5$ and $g(x) = x^2 - 3x + 4$, find each of the following:

(a) $(f + g)(x)$

Solution: $2x - 5 + x^2 - 3x + 4 = x^2 - x - 1$

(b) $\left(\frac{f}{g}\right)(x)$

Solution: $\frac{2x - 5}{x^2 - 3x + 4}$

(c) $(f \cdot g)(x)$

Solution:

$$\begin{aligned}(2x - 5)(x^2 - 3x + 4) &= 2x^3 - 6x^2 + 8x - 5x^2 + 15x - 20 \\ &= 2x^3 - 11x^2 + 23x - 20\end{aligned}$$

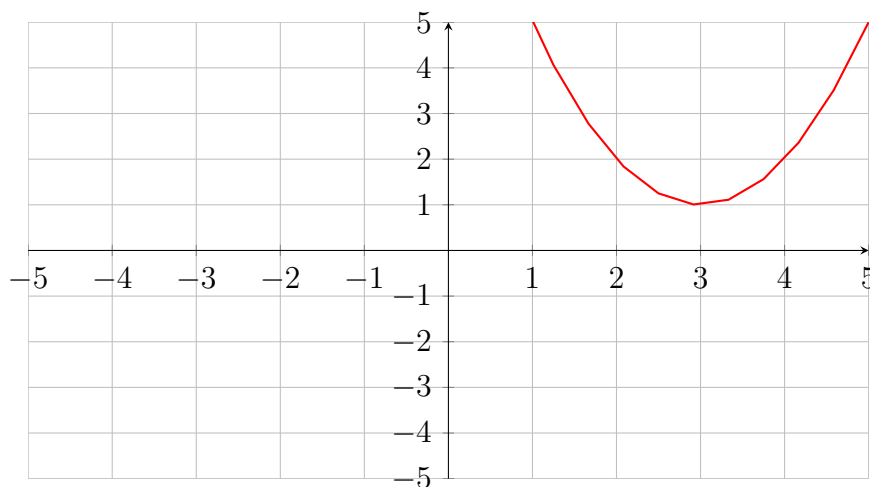
(d) $(f \circ g)(x)$

Solution:

$$\begin{aligned}f(g(x)) &= f(x^2 - 3x + 4) \\ &= 2(x^2 - 3x + 4) - 5 \\ &= 2x^2 - 6x + 8 - 5 \\ &= 2x^2 - 6x + 3\end{aligned}$$

33. Sketch the graph of $f(x) = (x - 3)^2 + 1$

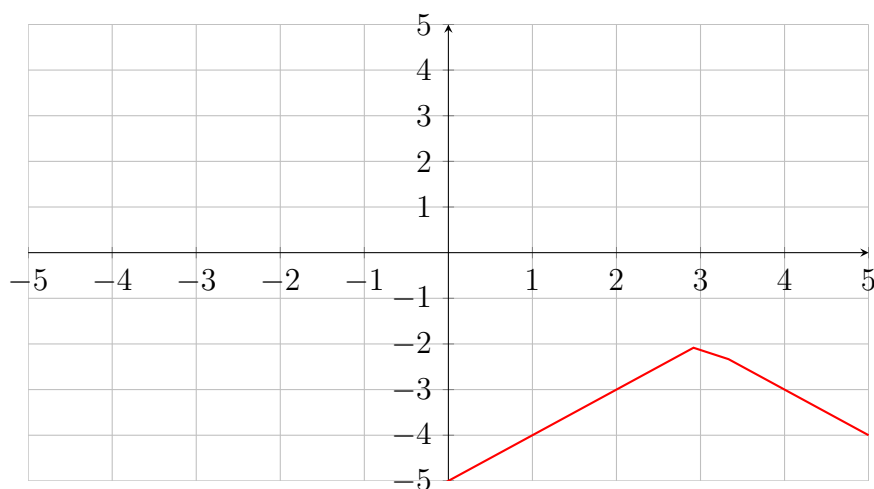
Solution: The vertex is $(3, 1)$. The graph has a horizontal shift of 3 and a vertical



shift of 1.

34. Sketch the graph of $f(x) = -|x - 3| - 2$

The graph is reflected across the x-axis, has a horizontal shift of 3 and a vertical shift of 2.



Solution:

35. A cab company charges a \$2 pick up fee and \$1.25 per mile. If the cab fare was \$13.25, how far was the trip?

Solution: cost = fee + cost per mile * miles

$$c = 2 + 1.25 * m$$

Then, as $c = 13.25$

$$13.25 = 2 + 1.25 * m$$

$$11.25 = 1.25 * m$$

$$9 = m$$

$$9 \text{ miles}$$

36. Find the inverse function of $y = e^x$

Solution:

$$y = e^x$$

$$x = e^y$$

$$\ln x = y$$

$$\ln x = f^{-1}(x)$$

37. Find the inverse function of $y = \frac{1}{x-2}$

Solution:

$$y = \frac{1}{x-2}$$

$$x = \frac{1}{y-2}$$

$$x(y-2) = 1$$

$$(y-2) = \frac{1}{x}$$

$$y = \frac{1}{x} + 2$$

$$f^{-1}(x) = \frac{1}{x} + 2$$

38. Decide if each of the following functions are one-to-one:

(a) $f(x) = x^2 - 2$

Solution: Nope

(b) $f(x) = x - 3$

Solution: yep

(c) $f(x) = \sqrt{x - 9}$

Solution: yup

39. Find the x-intercept(s) of $y = \ln(x - 3) + 1$

Solution:

$$\begin{aligned}0 &= \ln(x - 3) + 1 \\-1 &= \ln(x - 3) \\-1 &= \log_e(x - 3) \\e^{-1} &= x - 3 \\e^{-1} + 3 &= x\end{aligned}$$

40. Find the sum of the first 20 numbers in the sequence $\{1, 2, 3, 4, 5, \dots\}$

Solution: This is an arithmetic sequence with $a_n = 1 + 1(n - 1)$. Then the 20th term will be 20 and the first term is 1, so the sum is $S_{20} = \frac{20(20+1)}{2} = 210$

41. Find the first 3 terms of the sequence given by $a_n = 2a_{n-1} - 1$ for $n \geq 2$ and with $a_1 = 2$

Solution: $a_1 = 2$
 $a_2 = 4 - 1 = 3$
 $a_3 = 6 - 1 = 5$

42. y varies directly as x and y is equal to 10 when x is equal to 30. What is y when x is -7?

Solution:

$$\begin{aligned}y &= kx \\10 &= k30 \\ \frac{1}{3} &= k \\ y &= \frac{1}{3}x \\ y &= \frac{1}{3} * -7 \\ &= \frac{-7}{3}\end{aligned}$$

43. Find the exact value of each of the following:

(a) $\ln e^{.005}$

Solution: .005

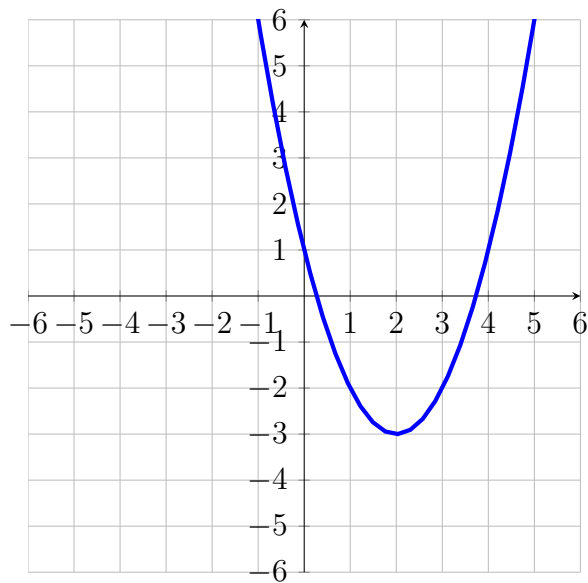
(b) $\log_2 \frac{1}{8}$

Solution: $\log_2 (2^{-3}) = -3$

(c) $e^{\log_e 5}$

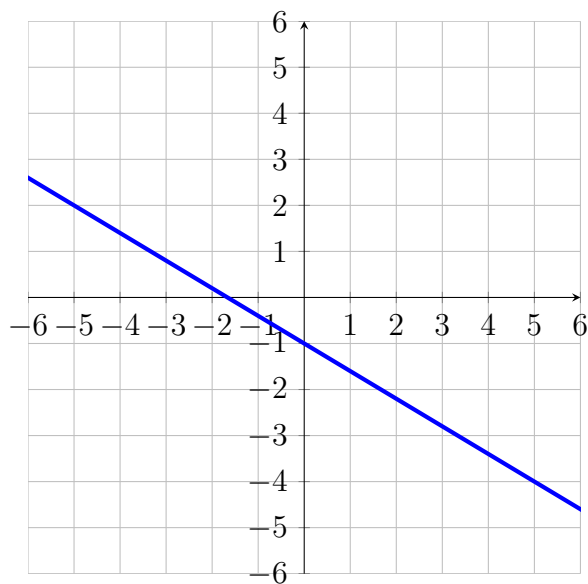
Solution: 5

44. Given the graph below, write the formula for the graphed function:



(a)

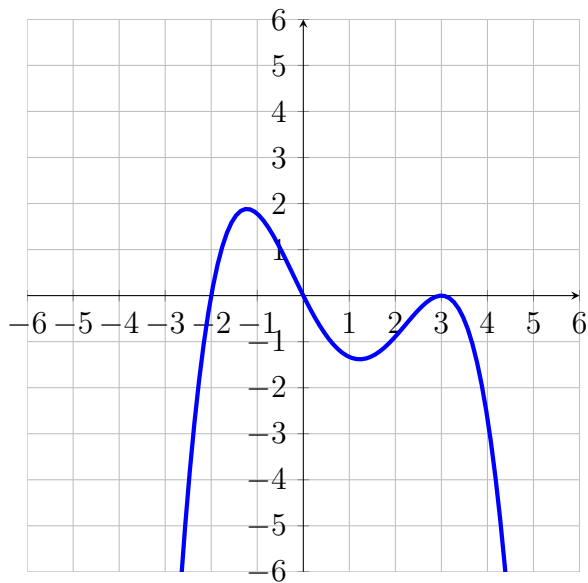
Solution: $y = (x - 2)^2 - 3$



(b)

Solution: $y = -\frac{3}{5}x - 1$

45. Use the zeros and their multiplicities to write an expression of the polynomial of least degree:



Solution: $y = -(x + 2)(x - 3)^2x$ (the actual graph has a lead coefficient of $1/9$ to scale it to fit in the grid)