

# Exam 4 Study Guide

Name \_\_\_\_\_

Answer the questions in the spaces provided. Feel free to use another piece of paper for your work

1. Find the work done:

- (a) A spherical water tank,  $24_{ft}$  in diameter, sits atop a  $60_{ft}$  tower. The tank is filled by a hose attached to the bottom of the sphere. If a 1.5 horsepower pump is used to deliver water up to the tank, how long will it take to fill the tank? (One horsepower is  $550_{ft-lbs}$  of work per second.)
- (b) An aquarium  $2_m$  long,  $1_m$  wide, and  $1_m$  deep is full of water. Find the work needed to pump half of the water out of the aquarium. (Use the fact that the density of water is  $1000_{kg/m^3}$ .)



2. Find the work done:

- (a) A spring has a natural length of  $40_{cm}$ . If a  $60_N$  force is required to keep the spring compressed  $10_{cm}$ , how much work is done during this compression? How much work is required to compress the spring to a length of  $25_{cm}$ ?

- (b) When a particle is located a distance  $x$  meters from the origin, a force of  $\cos\left(\frac{\pi x}{3}\right)$  newtons acts on it. How much work is done in moving the particle from  $x = 1$  to  $x = 2$ ? Interpret your answer by considering the work done from  $x = 1$  to  $x = 1.5$  and from  $x = 1.5$  to  $x = 2$ .



3. Find the average value of the functions on the given interval:

(a)  $f(x) = 3x^2 - 8x + \frac{x}{\sqrt{x^2+1}}$  on  $[3, 5]$   
(Hint: use substitution)

(b)  $g(x) = \cos^4(x)\sin(x)$  on  $[0, \pi]$

(c)  $h(x) = \cos^2(x) + 2x\sin(x^2) + \frac{4}{x^2}$  on  $[0, \pi]$   
(Hint: use substitution)



4. Find the limits:

(a)  $\lim_{x \rightarrow \infty} e^{-x^2}$

(b)  $\lim_{x \rightarrow -\infty} \frac{e^{3x} - e^{2x}}{e^{3x} + e^{2x}}$

(c)  $\lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{e^{3x} + e^{2x}}$





5. Find the derivative of the following functions:

(a)  $f(x) = e^{-x^2} + (x^2 - 3x)e^x$

(b)  $g(x) = \frac{e^{3x} - e^{2x}}{e^{3x} + e^{2x}}$

(c)  $h(x) = \frac{\cos(2x)}{e^{2\cos(x)}}$

(d)  $f(x) = \tan(1 + e^{2\cos(x)})$



6. Find the inverse functions for the following functions:

(a)  $f(x) = 3^{-x+4}$

(b)  $g(x) = \ln(5x)$

(c)  $h(x) = e^{-x^3}$

(d)  $f(x) = \frac{1-e^{-x}}{1+e^{-x}}$



7. Evaluate the following integrals:

(a)  $\int_0^1 e^x - x^e dx$

(b)  $\int \frac{e^x - e^{-x}}{2} dx$

(c)  $\int_0^1 (3x^2 - 4)e^{-x^3+4x} dx$

(Hint: use substitution and don't forget to change your bounds when you do)



8. Find the derivatives of the following functions:

(a)  $f(x) = \ln\left(\frac{\cos(x)}{x}\right)$

(b)  $g(x) = \ln\left(e^{3x^2 - \tan(x) + 3}\right)$

(c)  $h(x) = x \ln(x^2) + x^{\ln(x)}$

(d)  $f(x) = \frac{x}{\ln(x)}$





9. Find the derivatives of the following functions:

(a)  $f(x) = \sin^{-1}(1 - x^2)$

(b)  $g(x) = -\cot^{-1}(x + x^2)$

(c)  $h(x) = x \cos^{-1}(x)$

(d)  $f(x) = \cos^{-1}(x) \sin^{-1}(x)$



10. Evaluate the integrals:

Hint- Don't forget to use substitution

(a)  $\int \frac{dx}{\sqrt{x(1+x)}}$

(b)  $\int \frac{\sin(x)}{1+\cos^2(x)} dx$

(c)  $\int \frac{e^{2x}}{\sqrt{1-e^{4x}}} dx$



11. Solve using l'Hospital:

(a)  $\lim_{x \rightarrow \infty} \frac{\ln(x)}{\sqrt{x}}$

(b)  $\lim_{x \rightarrow \infty} \frac{x+x^2}{1-x^{7/3}}$

(c)  $\lim_{x \rightarrow 1^+} \ln([x^7 - 1] - [x^5 - 1])$

(d)  $\lim_{x \rightarrow 0} (1+2x)^{1/x}$

(e)  $\lim_{x \rightarrow \infty} x e^{-x}$