Exam 3 Study Guide

Name______________________________

Answer the questions in the spaces provided. Feel free to use another piece of paper for your work

1. Evaluate the following integrals:

   (a) $\int_0^1 4x^4 + 5 - \frac{2}{x^2} - \cos(x)dx$

   (b) $\int \cos(x)\sin(x)dx$

   (c) $\int_0^{\pi/2} \sin(t/3)dt$

   (d) $\int_0^x \frac{1}{\sqrt{4t}}dt$

2. Use the following to solve 2(c) - 2(d)

The substitution rule says,

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

(a) **Example:**
$\int_0^4 \sqrt{2x+1}dx$

Then $f(u) = \sqrt{u}$, $u = 2x+1$, and $\frac{du}{dx} = 2 \implies \frac{du}{2} = dx$, so

$\int_0^4 \sqrt{2x+1}dx \implies \int_{2(0)+1}^{2(4)+1} \sqrt{u}\ \frac{du}{2} = \frac{1}{2}\int_1^9 \sqrt{u}\ du = \frac{1}{2}(\frac{2}{3}u^{\frac{3}{2}})\Big]_1^9 = \frac{1}{3}u^{\frac{3}{2}}\Big]_1^9 = \frac{1}{3}9^{\frac{3}{2}} - \frac{1}{3}1^{\frac{3}{2}} = \frac{26}{3}$

(b) **Example:**
$\int_{-\frac{1}{2}}^{\frac{-10+\sqrt{16\pi+36}}{8}} \cos\left(2t^2+5t+2\right)(4t+5)dt$

Then $f(u) = \cos(u)$, $u = 2t^2+5t+2 = g(t)$, and $\frac{du}{dt} = 4t+5 \implies du = (4t+5)dt$, so since $g(-\frac{1}{2}) = 0$ and $g(\frac{-10+\sqrt{16\pi+36}}{8}) = \frac{\pi}{2}$ we get

$\int_{-\frac{1}{2}}^{\frac{-10+\sqrt{16\pi+36}}{8}} \cos\left(2t^2+5t+2\right)(4t+5)dt \implies \int_0^{\pi/2} \cos(u)du$

(c) $\int_0^{\pi/9} \frac{\sin(\sqrt{x})}{-2\sqrt{x}}dx$

(d) $\int \sec^3(y)\tan(y)dy$

3. Determine if the following statements are true:

(a) $\int_{-1}^{1}\left(x^5 - 6x^9 + \frac{\sin x}{(1+x^4)^2}\right)dx = 0$

(b) If f is continuous on $[a, b]$, then (pay attention to the limits of integration)

$$\int_b^a 5f(x)dx = -5\int_a^b f(x)dx$$

(c) $\int_{\pi}^{2\pi} \frac{\sin x}{x} dx = \int_{\pi}^{3\pi} \frac{\sin x}{x} dx + \int_{3\pi}^{2\pi} \frac{\sin x}{x} dx$

(d) $\int_0^2 (x - x^3)\, dx$ represents the area under the curve $y = x - x^3$ from 0 to 2

(e) $\int_{-2}^{1} \frac{1}{x^4} dx = -\frac{3}{8}$

4. Find the area enclosed by the curves:

(a) $y = 12 - x^2$ and $y = x^2 - 6$?

(b) $f(t) = \sec^2(t)$, $g(t) = 8\cos(t)$, and bounded by $-\pi/3 \leq x \leq \pi/3$

(c) $h(x) = \sqrt{x-1}$, $x - y = 1$

(d) $y = x^4$, $y = 2 - |x|$

5. Find the volume of a solid given the function for the cross-sectional area $A(x)$ on the prescribed interval:

   (a) $A(x) = x^2 - x + 13$ and $3 \le x \le 6$?

   (b) $A(x) = \frac{2x}{x^2+6}$ and $2 \le x \le 4$

6. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line:

   (a) $f(x) = x^2 - x + 13$, $3 \le x \le 6$, rotated about the x-axis?

   (b) $y = 1 + \sec(x)$, $y = 3$, rotated about $y = 1$

7. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line:

(a) $x^2 + y^2 = a^2$, $x = a + h$ (where $a \geq 0$, $h \geq 0$); about the y-axis

(b) $y = cos^2(x)$, $|x| \leq \pi/2$, $y = 1/4$ about $x = \pi/2$

(c) $xy = 1$, $x = 0$, $y = 1$ about $y = 3$

(d) $x + y = 4$, $x = y^2 - 4y + 4$ about $x = 0$

8. State the Fundemental Theorem of Calculus:

   (a) Part(1); about the y-axis

   (b) Part(2)

9. State the formula for:

   (a) Disk Method

   (b) Washer Method

   (c) Shell Method

   (d) The "limit/series" definition of the integral

10. State 3 properties of the integral:

(a)

(b)

(c)

11. Use the Midpoint Rule pproximate the area of the function, for the stated interval, and with the stated number of rectangles:

(a) $f(x) = (x+4)^2$, the interval $a = 1$ to $b = 5$, and $n = 2$

(b) $g(x) = (x+4)^2$, the interval $a = 1$ to $b = 5$, and $n = 4$

(c) $f(x) = (x+4)^2$, the interval $a = 1$ to $b = 5$, and $n \to \infty$

12. Find the area between the given curves on the stated interval:

   (a) $f(x) = (x+4)^2$ and $g(x) = (x+4)^3$ on the interval $a = 1$ to $b = 5$

   (b) $h(x) = \cos^2(x)$ and $j(x) = \sin^2(x)$ on the interval $a = \pi/4$ to $b = \pi/2$

13. What is the key difference in solutions given by the antiderivative, the indefinite integrl, and the definite integral?

   (a)