Calc I

Day 11

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Derivatives

Def(Secant Line):

A secant line of a function is a line which goes through at least two points of the function.

Example(Secant Line):

Let's say we have a function, $f(x) = 2x^2$. One of its secant lines would be the line going through the points:

$$(1, f(1)) = (1, 2) \text{ and } (2, f(2)) = (2, 8)$$

Since we know the points this particular line goes through, we can use the point-slope formula($y - y_0 = m(x - x_0)$) to find its equation.

Example(Secant Line):

First we need to find its slope, m.

Since our line goes through the points

$$(1, f(1)) = (1, 2) \text{ and } (2, f(2)) = (2, 8)$$

we know the slope is:

$$m = \frac{\Delta y}{\Delta x} = \frac{f(2) - f(1)}{2 - 1} = \frac{8 - 2}{2 - 1} = \frac{6}{1} = 6 \, .$$

Example(Secant Line):

Now that we know the slope we need to chose one of the two points and plug everything into the point-slope formula.

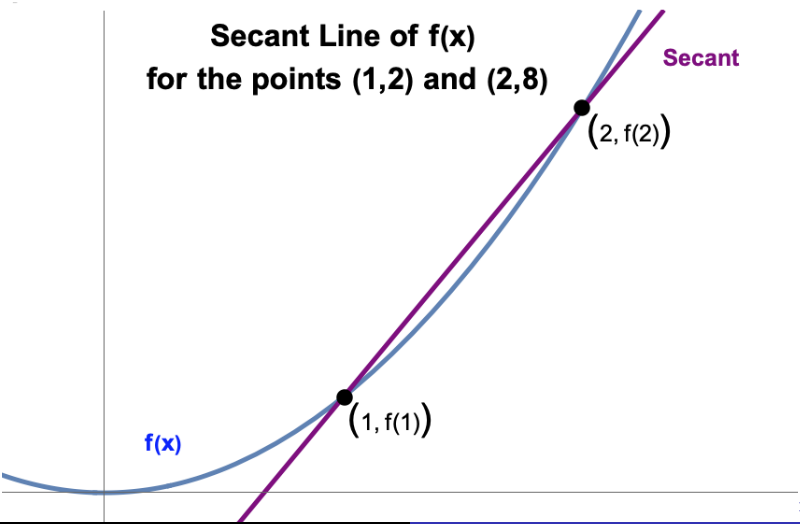
Let's use the point $(2, 8)$. Then plugging everything in we get,

$$y - 8 = 6(x - 2)$$

Solving for y we get the equation of our line,

$$y = 6x - 4$$

Example(Secannt Line):



Question:

Let's say we have a function, $f(x)$ and a secant line that goes through two points. Wht happens if we fix one of those points, say $x = a$ and move the other one, $a \pm h$, to it?

So our secant line would be the line through:

$$(a, f(a)) \text{ and } (a \pm h, f(a \pm h))$$

Answer:

Our secant line would ”approach” the tangent line of $f(x)$ at $x = a$

Def(Tangent Line):

A tangent line of a function is a line which ”touches” at least one point of the function, and has the same slope as the function at that point.

Example(Tangent Line):

Let's use the same function as before, $f(x) = 2x^2$. And let's consider the line that "lays tagent to $f(x)$ at $x = 1$."

$$(1, f(1)) = (1, 2)$$

Since we know the point this particular line goes through, we can use the point-slope formula($y - y_0 = m(x - x_0)$) to find the equation that describes it.

Example(Tangent Line):

First we need to find its slope, m.

Since our line goes through the point

$$(1, f(1)) = (1, 2)$$

and has the same slope as our function $f(x)$ at $x = 1$, we know the slope is:

$$m = \lim_{h \to 0} \frac{f(1+h) - f(1)}{(1+h) - (1)} = \lim_{h \to 0} \frac{(2(1+h)^2) - (2(1)^2)}{h}$$

$$= \lim \frac{(2h^2 + 4h + 2) - (2)}{h} = \lim \frac{(2h^2 + 4h)}{h} = \lim (2h + 4) = 4$$

Example(Tangent Line):

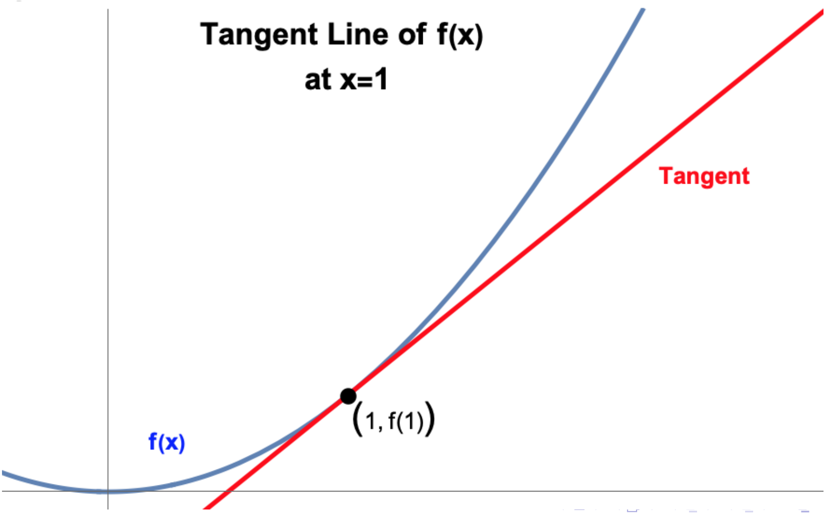
Now that we know the slope we need to plug everything into the point-slope formula, to get.

$$y - 2 = 4(x - 1)$$

Solving for y we get the equation of our line,

$$y = 4x - 2$$

Example(Tangent Line):



The slope of the tangent line of our function at $x = a$ is also the slope of our function at $x = a$.

We call the slope of our function at $x = a$, the *derivative* of our function at $x = a$

The Limit Definition of the Derivative of $f(x)$ at a point $x = a$:

Given a function $f(x)$, its derivative at $x = a$ is,

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = f'(a)$$

We can generalize this to all points in our domain by ”exchanging” a with x. And in doing so, we will have derived the function describing the derivative of our function.

The Limit Definition of the Derivative of $f(x)$:

Given a function $f(x)$, its derivative is,

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

Def(Differentiable at a Point $x = a$):

Def(Differentiable on an Interval):

The Derivative Rules:

1.
2.
3.
4.
5.
6.

Suggested Readings:

Calculus 8th edition by James Stewart:

- Chapter 3

The Calculus Story A Mathematical Adventure by David Acheson:

- Chapters 3-6, Chapters 11-12, and Chapters 16-18

The Cartoon Guide to Calculus by Larry Gonick:

- Chapters 2-7