Calc I

Day 2
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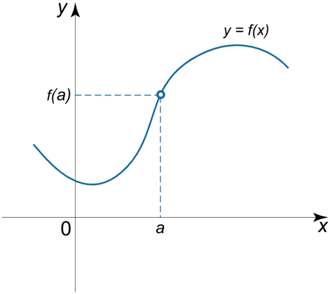
Continuity of Functions

Imagine I gave you a function and its graph(like the one below); and, I ask you to give me the value of $f(a)$.

So you look and find that its graph has a hole at a.



What should you tell me?

Definition (**Discontinuous Functions**)

We say a function $f(x)$ is discontinuous if it's not continuous.

Wait... isn't that a circular definition??? Thanks for the help bro!

Definition (**Continuous at a point** a)

We say a function $f(x)$ is continuous at a point a if $\lim_{x \to a} f(x) = f(a)$.

I'll explain that $\lim_{x \to a} f(x)$ notation shortly but for now... Let's just understand what this all means...

One way to understand what this means is by looking at a handful of graphs.

Examples:

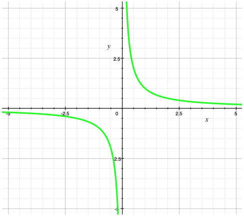


Figure: $f(x) = \frac{1}{x}$

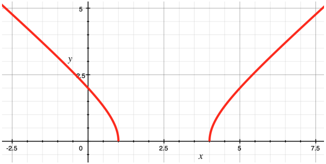


Figure: $f(x) = \sqrt{x^2 - 5x + 4}$

Examples:

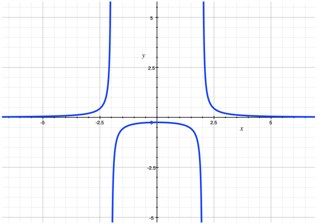


Figure: $f(x) = \frac{1}{x^2-4}$

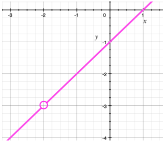


Figure: $f(x) = \frac{x^2+x-2}{x+2}$

Examples:

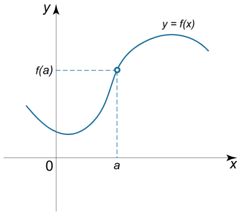
$f(x) = \frac{1}{x}$ $f(0) = DNE$ so it fails to be continuous.

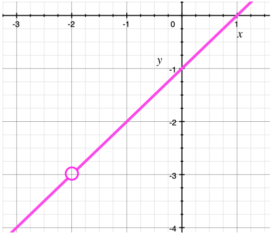
$f(x) = \sqrt{x^2 - 5x + 4}$ $f(a) = DNE$ for $1 < a < 4$ so it fails to be continuous.

$f(x) = \frac{1}{x^2-4}$ $f(2) = DNE$ and $f(-2) = DNE$ so it fails to be continuous.

$f(x) = \frac{x^2+x-2}{x+2}$ $f(-2) = DNE$ so it fails to be continuous.

Now, looking at the first and the last graph presented, it's natural to ask:





If those holes weren't there, what would the function be???

The Limit

The limit is a mathematical tool that allows us to answer the question,

If I could reach a, what would $f(a)$ be?

It is written,

$$\lim_{x \to a} f(x)$$

and is read,

"the limit of $f(x)$ as x approaches a."

Example:

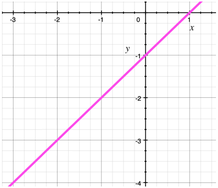


Figure: $\lim_{x \to -2} f(x) = -4$

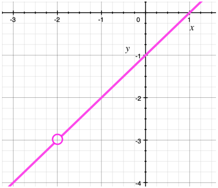


Figure: $\lim_{x \to -2} g(x) = -4$

Example:

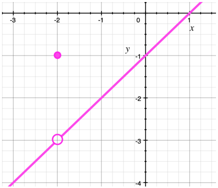


Figure: $\lim_{x \to -2} h(x) = -4$

So... How do we solve these?

Intuitively we may assume we can either,

- try to close the discontinuity algebraically, or
- use a table

Example:

Let $f(x) = \frac{sin(x)}{x}$, with a discontinuity at $x = 0$

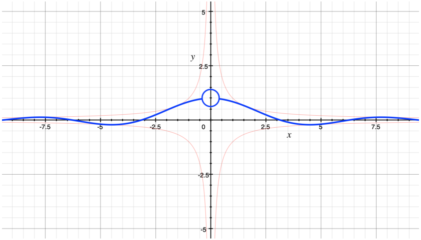


Figure: The graph of $\frac{\sin(x)}{x}$ in blue bounded by $\pm\frac{1}{x}$ in red

We want to find $\lim_{x\to 0} f(x)$

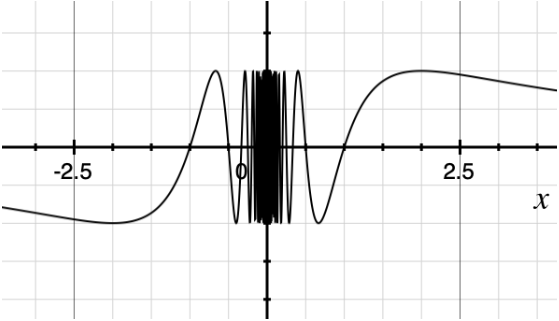
Setup:

Let's begin by looking at a table of values near our discontinuity at $x = 0$:

x	$f(x)$
± 1	0.84147098
± 0.5	0.95885108
± 0.1	0.99833417
± 0.01	0.99998333
± 0.001	0.99999983

Solution: From this, we might correctly conclude $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

Let's now assume we're instead given $g(x) = \sin\left(\frac{\pi}{x}\right)$ and asked to solve $\lim_{x\to 0} g(x)$.



If we played the same trick here, we would find that it doesn't matter how small we let our changes in x get, things $f(x)$ never seems to "focus" in on a single number.

One-Sided Limit

—–SLIDE 3—–

Limit Laws and Putting Limits to Use

—-SLIDE 3—-

Continuity of Functions Revisited

Definition (**Continuous at a point** a)

We say a function $f(x)$ is continuous at a point a if $\lim_{x \to a} f(x) = f(a)$.

Calculus I 8th edition by James Stewart:

- Chapter 1.8, 1.5-1.7, and 1.4

The Calculus Story A Mathematical Adventure by David Acheson

- Pg. 117, 142, Chapters 25, 26, 3, 4

The Cartoon Guide to Calculus by Larry Gonick

- Chapter -1 and 1