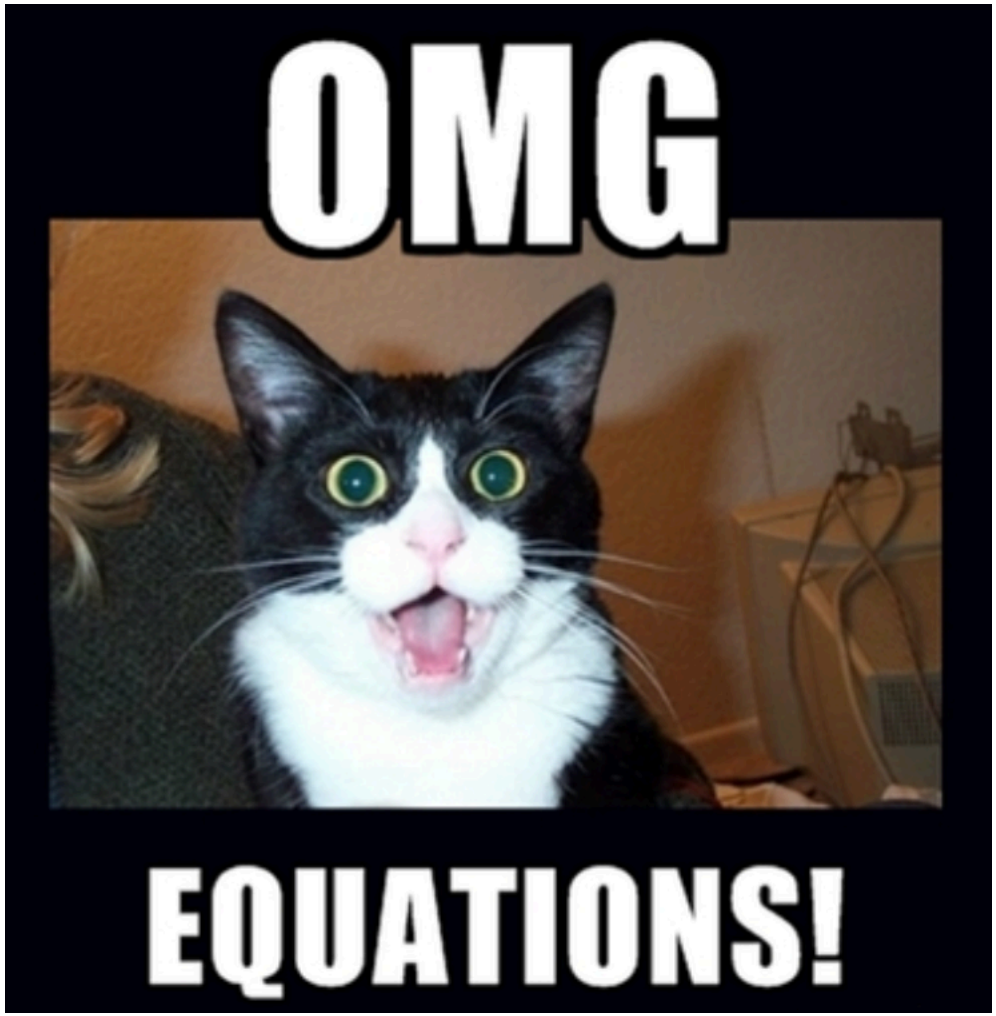
Day 9



An **equation** is a statement that indicates that two expressions are equal. The following are equations.

$$x = 5 \qquad y + 2 = 12 \qquad -4z = 28$$

All equations have an equal sign. Furthermore, notice that the equal sign separates the equation into two parts, the left-hand side and the right-hand side. A **solution to an equation** is a value of the variable that makes the equation a true statement. Substituting a solution into an equation for the variable makes the right-hand side equal to the left-hand side.

Equation	Solution	Check	
$x = 5$	5	$x = 5$ $\downarrow$ $5 = 5$ ✓	Substitute 5 for x. Right-hand side equals left-hand side.
$y + 2 = 12$	10	$y + 2 = 12$ $\downarrow$ $10 + 2 = 12$ ✓	Substitute 10 for y. Right-hand side equals left-hand side.
$-4z = 28$	-7	$-4z = 28$ $\downarrow$ $-4(-7) = 28$ ✓	Substitute -7 for z. Right-hand side equals left-hand side.

Avoiding Mistakes

Be sure to notice the difference between solving an equation versus simplifying an expression. For example, $2x + 1 = 7$ is an equation, whose solution is 3, while $2x + 1 + 7$ is an expression that simplifies to $2x + 8$.

Determine whether the given number is a solution to the equation.

a. $4x + 7 = 5$; $-\frac{1}{2}$

b. $-4 = 6w - 14$; 3

The set of all solutions to an equation is called the **solution set** and is written with set braces. For example, the solution set for **Example 1(a)** is $\left\{-\frac{1}{2}\right\}$.

Definition of a Linear Equation in One Variable

Let a, b, and c be real numbers such that $a \neq 0$. A **linear equation in one variable** is an equation that can be written in the form

$$ax + b = c$$

Note: A linear equation in one variable is often called a first-degree equation because the variable x has an implied exponent of 1.

Examples	**Notes**
$2x + 4 = 20$	$a = 2$, $b = 4$, $c = 20$
$-3x - 5 = 16$ can be written as $-3x + (-5) = 16$	$a = -3$, $b = -5$, $c = 16$
$5x + 9 - 4x = 1$ can be written as $x + 9 = 1$	$a = 1$, $b = 9$, $c = 1$

If two equations have the same solution set, then the equations are equivalent. For example, the following equations are equivalent because the solution set for each equation is {6}.

Equivalent Equations		Check the Solution 6		
$2x - 5 = 7$	$\longrightarrow$	$2(6) - 5 \stackrel{?}{=} 7$	$\Rightarrow$	$12 - 5 \stackrel{?}{=} 7$ ✓
$2x = 12$	$\longrightarrow$	$2(6) \stackrel{?}{=} 12$	$\Rightarrow$	$12 \stackrel{?}{=} 12$ ✓
$x = 6$	$\longrightarrow$	$6 \stackrel{?}{=} 6$	$\Rightarrow$	$6 \stackrel{?}{=} 6$ ✓

Multiplication and Division Properties of Equality

Let a, b, and c represent algebraic expressions, $c \neq 0$.

1. Multiplication property of equality: If $a = b$, then $ac = bc$

2. *Division property of equality: If $a = b$ then $\frac{a}{c} = \frac{b}{c}$

Solve the equations using the division property of equality.

a. $12x = 60$

b. $48 = -8w$

c. $-x = 8$

Solution:

a. $12x = 60$

$$\frac{12x}{12} = \frac{60}{12}$$ To obtain a coefficient of 1 for the x-term, divide both sides by 12.

$1x = 5$ Simplify.

$x = 5$ Check: $12x = 60$

$12(5) \stackrel{?}{=} 60$

The solution set is $\{5\}$. $60 \stackrel{?}{=} 60$ ✓ True

b. $48 = -8w$

$$\frac{48}{-8} = \frac{-8w}{-8}$$ To obtain a coefficient of 1 for the w-term, divide both sides by -8.

$-6 = 1w$ Simplify.

$-6 = w$ Check: $48 = -8w$

$48 \stackrel{?}{=} -8(-6)$

The solution set is $\{-6\}$. $48 \stackrel{?}{=} 48$ ✓ True

c. $-x = 8$ Note that $-x$ is equivalent to $-1 \cdot x$.

$-1x = 8$

$$\frac{-1x}{-1} = \frac{8}{-1}$$ To obtain a coefficient of 1 for the x-term, divide by -1.

$x = -8$ Check: $-x = 8$

$-(-8) \stackrel{?}{=} 8$

The solution set is $\{-8\}$. $8 \stackrel{?}{=} 8$ ✓ True

Solve the equation by using the multiplication property of equality.

$$-\frac{2}{9}q = \frac{1}{3}$$

Solution:

$$-\frac{2}{9}q = \frac{1}{3}$$

$$\left(-\frac{9}{2}\right)\left(-\frac{2}{9}q\right) = \frac{1}{3}\left(-\frac{9}{2}\right)$$

To obtain a coefficient of 1 for the q-term, multiply by the reciprocal of $-\frac{2}{9}$, which is $-\frac{9}{2}$.

$$1q = -\frac{3}{2}$$

Simplify. The product of a number and its reciprocal is 1.

$$q = -\frac{3}{2}$$

Check: $-\frac{2}{9}q = \frac{1}{3}$

$$-\frac{2}{9}\left(-\frac{3}{2}\right) \stackrel{?}{=} \frac{1}{3}$$

$$\frac{1}{3} \stackrel{?}{=} \frac{1}{3} \checkmark \quad \text{True}$$

The solution set is $\left\{-\frac{3}{2}\right\}$.

Solve the equation by using the division property of equality.

$$-3.43 = -0.7z$$

Solution:

$$-3.43 = -0.7z$$

$$\frac{-3.43}{-0.7} = \frac{-0.7z}{-0.7}$$ To obtain a coefficient of 1 for the z-term, divide by -0.7.

$$4.9 = 1z$$ Simplify.

$$4.9 = z$$

$$z = 4.9$$

Check: $-3.43 = -0.7z$

$-3.43 \stackrel{?}{=} -0.7(4.9)$

$-3.43 \stackrel{?}{=} -3.43$ ✓ True

The solution set is $\{4.9\}$.

Solve the equation by using the multiplication property of equality.

$$\frac{d}{6} = -4$$

Solution:

$\frac{d}{6} = -4$	
$\frac{1}{6}d = -4$	$\frac{d}{6}$ is equivalent to $\frac{1}{6}d$.
$\frac{6}{1} \cdot \frac{1}{6}d = -4 \cdot \frac{6}{1}$	To obtain a coefficient of 1 for the d-term, multiply by the reciprocal of $\frac{1}{6}$, which is $\frac{6}{1}$.
$1d = -24$	Simplify.
$d = -24$	<u>Check</u>: $\frac{d}{6} = -4$
	$\frac{-24}{6} \stackrel{?}{=} -4$
The solution set is $\{-24\}$.	$-4 \stackrel{?}{=} -4$ ✓ True

Linear Equations

Example 1 Solving a Linear Equation

Solve the equation. $-2w - 7 = 11$

Solution:

$$-2w - 7 = 11$$

$$-2w - 7 + 7 = 11 + 7$$ Add 7 to both sides of the equation. This isolates the w-term.

$$-2w = 18$$

$$\frac{-2w}{-2} = \frac{18}{-2}$$ Next, apply the division property of equality to obtain a coefficient of 1 for w. Divide by -2 on both sides.

$$w = -9$$

Check:

$$-2w - 7 = 11$$

$$-2(-9) - 7 \stackrel{?}{=} 11$$ Substitute $w = -9$ in the original equation.

$$18 - 7 \stackrel{?}{=} 11$$

$$11 \stackrel{?}{=} 11 \checkmark$$ True

The solution set is $\{-9\}$.

Skill Practice Solve the equation.

1. $-5y - 5 = 10$

Answer

Example 2 Solving a Linear Equation

Solve the equation. $2 = \frac{1}{5}x + 3$

Solution:

$$2 = \frac{1}{5}x + 3$$

$$2 - 3 = \frac{1}{5}x + 3 - 3$$ Subtract 3 from both sides. This isolates the x-term.

$$-1 = \frac{1}{5}x$$ Simplify.

$$5(-1) = 5 \cdot \left(\frac{1}{5}x\right)$$ Next, apply the multiplication property of equality to obtain a coefficient of 1 for x.

$$-5 = 1x$$

$$-5 = x$$ Simplify. The answer checks in the original equation.

The solution set is {–5}.

Skill Practice Solve the equation.

2. $2 = \frac{1}{2}a - 7$

Answer

Example 3 **Solving a Linear Equation**

Solve the equation. $6x - 4 = 2x - 8$

Solution:

$6x - 4 = 2x - 8$	
$6x - 2x - 4 = 2x - 2x - 8$	Subtract $2x$ from both sides leaving $0x$ on the right-hand side.
$4x - 4 = 0x - 8$	Simplify.
$4x - 4 = -8$	The x-terms have now been combined on one side of the equation.
$4x - 4 + 4 = -8 + 4$	Add 4 to both sides of the equation. This combines the constant terms on the *other* side of the equation.
$4x = -4$	
$\dfrac{4x}{4} = \dfrac{-4}{4}$	To obtain a coefficient of 1 for x, divide both sides of the equation by 4.
$x = -1$	The answer checks in the original equation.

The solution set is $\{-1\}$.

Skill Practice Solve the equation.

3. $10x - 3 = 4x - 2$

Answer

Solving a Linear Equation in One Variable

Step 1 Simplify both sides of the equation.

- Clear parentheses
- Combine *like* terms

Step 2 Use the addition or subtraction property of equality to collect the variable terms on one side of the equation.

Step 3 Use the addition or subtraction property of equality to collect the constant terms on the other side of the equation.

Step 4 Use the multiplication or division property of equality to make the coefficient of the variable term equal to 1.

Step 5 Check your answer.

Avoiding Mistakes

When distributing a negative number through a set of parentheses, be sure to change the signs of every term within the parentheses.

Solve the equation. $7 + 3 = 2(p - 3)$

Solution:

$7 + 3 = 2(p - 3)$

$10 = 2p - 6$ **Step 1:** Simplify both sides of the equation by clearing parentheses and combining *like* terms.

Step 2: The variable terms are already on one side.

$10 + 6 = 2p - 6 + 6$ **Step 3:** Add 6 to both sides to collect the constant terms on the other side.

$16 = 2p$

$\frac{16}{2} = \frac{2p}{2}$ **Step 4:** Divide both sides by 2 to obtain a coefficient of 1 for *p*.

$8 = p$ **Step 5:** Check:

$7 + 3 = 2(p - 3)$

$10 \stackrel{?}{=} 2(8 - 3)$

$10 \stackrel{?}{=} 2(5)$

$10 \stackrel{?}{=} 10 \checkmark$ True

The solution set is {8}.

Skill Practice Solve the equation.

4. $12 + 2 = 7(3 - y)$

Answer

Get into groups of two or three and…

Solve the equation. $2.2y - 8.3 = 6.2y + 12.1$

Solve the equation. $2.2y - 8.3 = 6.2y + 12.1$

Solution:

$$2.2y - 8.3 = 6.2y + 12.1$$

Step 1: The right- and left-hand sides are already simplified.

$$2.2y - 2.2y - 8.3 = 6.2y - 2.2y + 12.1$$
$$-8.3 = 4y + 12.1$$

Step 2: Subtract $2.2y$ from both sides to collect the variable terms on one side of the equation.

$$-8.3 - 12.1 = 4y + 12.1 - 12.1$$
$$-20.4 = 4y$$

Step 3: Subtract 12.1 from both sides to collect the constant terms on the other side.

$$\frac{-20.4}{4} = \frac{4y}{4}$$
$$-5.1 = y$$

Step 4: To obtain a coefficient of 1 for the y-term, divide both sides of the equation by 4.

$$y = -5.1$$

Step 5: Check:

$$2.2y - 8.3 = 6.2y + 12.1$$
$$2.2(-5.1) - 8.3 \stackrel{?}{=} 6.2(-5.1) + 12.1$$
$$-11.22 - 8.3 \stackrel{?}{=} -31.62 + 12.1$$
$$-19.52 \stackrel{?}{=} -19.52 \checkmark \text{ True}$$

The solution set is {–5.1}.

Solve the equation. $2 + 7x - 5 = 6(x + 3) + 2x$

Solution:

$2 + 7x - 5 = 6(x + 3) + 2x$	
$-3 + 7x = 6x + 18 + 2x$	**Step 1:** Add *like* terms on the left. Clear parentheses on the right.
$-3 + 7x = 8x + 18$	Combine *like* terms.
$-3 + 7x - 7x = 8x - 7x + 18$	**Step 2:** Subtract $7x$ from both sides.
$-3 = x + 18$	Simplify.
$-3 - 18 = x + 18 - 8$	**Step 3:** Subtract 18 from both sides.
$-21 = x$	**Step 4:** Because the coefficient of the x term is already 1, there is no need to apply the multiplication or division property of equality.
$x = -21$	
The solution set is $\{-21\}$.	**Step 5:** The check is left to the reader.

Skill Practice Solve the equation.

6. $4(2y - 1) + y = 6y + 3 - y$

[Answer]

Solving a Linear Equation in One Variable

Step 1 Simplify both sides of the equation.

- Clear parentheses
- Combine *like* terms

Step 2 Use the addition or subtraction property of equality to collect the variable terms on one side of the equation.

Step 3 Use the addition or subtraction property of equality to collect the constant terms on the other side of the equation.

Step 4 Use the multiplication or division property of equality to make the coefficient of the variable term equal to 1.

Step 5 Check your answer.

Avoiding Mistakes

When distributing a negative number through a set of parentheses, be sure to change the signs of every term within the parentheses.

Solve the equation. $9 - (z - 3) + 4z = 4z - 5(z + 2) - 6$

Solution:

$$9-(z-3)+4z = 4z-5(z+2)-6$$

$$9-z+3+4z = 4z-5z-10-6$$

Step 1: Clear parentheses.

Combine *like* terms.

$$12+3z = -z-16$$

$$12+3z+z = -z+z-16$$

Step 2: Add z to both sides.

$$12+4z = -16$$

$$12-12+4z = -16-12$$

Step 3: Subtract 12 from both sides.

$$4z = -28$$

$$\frac{4z}{4} = \frac{-28}{4}$$

Step 4: Divide both sides by 4.

$$z = -7$$

Step 5: The check is left for the reader.

The solution set is {–7}.

I. Conditional Equations

An equation that is true for some values of the variable but false for other values is called a **conditional equation.** The equation $x + 4 = 6$, for example, is true on the condition that $x = 2$. For other values of x, the statement $x + 4 = 6$ is false.

$$x + 4 = 6$$

$$x + 4 - 4 = 6 - 4$$

$$x = 2 \quad \text{(Conditional equation)} \quad \text{Solution set: } \{2\}$$

II. Contradictions

Some equations have no solution, such as $x + 1 = x + 2$. There is no value of x, that when increased by 1 will equal the same value increased by 2. If we try to solve the equation by subtracting x from both sides, we get the contradiction $1 = 2$. This indicates that the equation has no solution. An equation that has no solution is called a **contradiction**. The solution set is the empty set. The **empty set** is the set with no elements and is denoted by $\{ \}$.

$$x + 1 = x + 2$$

$$x - x + 1 = x - x + 2$$

$1 = 2$ (Contradiction) Solution set: $\{ \}$

Pro Tip

TIP: The empty set is also called the null set and can be expressed by the symbol $\varnothing$.

Avoiding Mistakes

There are two ways to express the empty set: { } or $\varnothing$. Be sure that you do not use them together. It would be incorrect to write $\{\varnothing\}$.

III. Identities

An equation that has all real numbers as its solution set is called an **identity**. For example, consider the equation, $x + 4 = x + 4$. Because the left- and right-hand sides are *identical*, any real number substituted for x will result in equal quantities on both sides. If we subtract x from both sides of the equation, we get the identity $4 = 4$. In such a case, the solution is the set of all real numbers.

$$x + 4 = x + 4$$

$$x - x + 4 = x - x + 4$$

$4 = 4$ (Identity) Solution set: The set of real numbers.

Solve the equation. Identify each equation as a conditional equation, a contradiction, or an identity.

a. $4k - 5 = 2(2k - 3) + 1$

b. $2(b - 4) = 2b - 7$

c. $3x + 7 = 2x - 5$

Solution:

a.

$$4k - 5 = 2(2k - 3) + 1$$

$$4k - 5 = 4k - 6 + 1 \qquad \text{Clear parentheses.}$$

$$4k - 5 = 4k - 5 \qquad \text{Combine } \textit{like} \text{ terms.}$$

$$4k - 4k - 5 = 4k - 4k - 5 \qquad \text{Subtract } 4k \text{ from both sides.}$$

$$-5 = -5 \quad \text{(Identity)}$$

This is an identity. Solution set: The set of real numbers.

b.

$$2(b - 4) = 2b - 7$$

$$2b - 8 = 2b - 7 \qquad \text{Clear parentheses.}$$

$$2b - 2b - 8 = 2b - 2b - 7 \qquad \text{Subtract } 2b \text{ from both sides.}$$

$$-8 = -7 \quad \text{(Contradiction)}$$

This is a contradiction. Solution set: { }

c.

$$3x + 7 = 2x - 5$$

$$3x - 2x + 7 = 2x - 2x - 5 \qquad \text{Subtract } 2x \text{ from both sides.}$$

$$x + 7 = -5 \qquad \text{Simplify.}$$

$$x + 7 - 7 = -5 - 7 \qquad \text{Subtract } 7 \text{ from both sides.}$$

$$x = -12 \quad \text{(Conditional equation)}$$

This is a conditional equation. The solution set is {–12}. (The equation is true only on the condition that $x = -12$.)