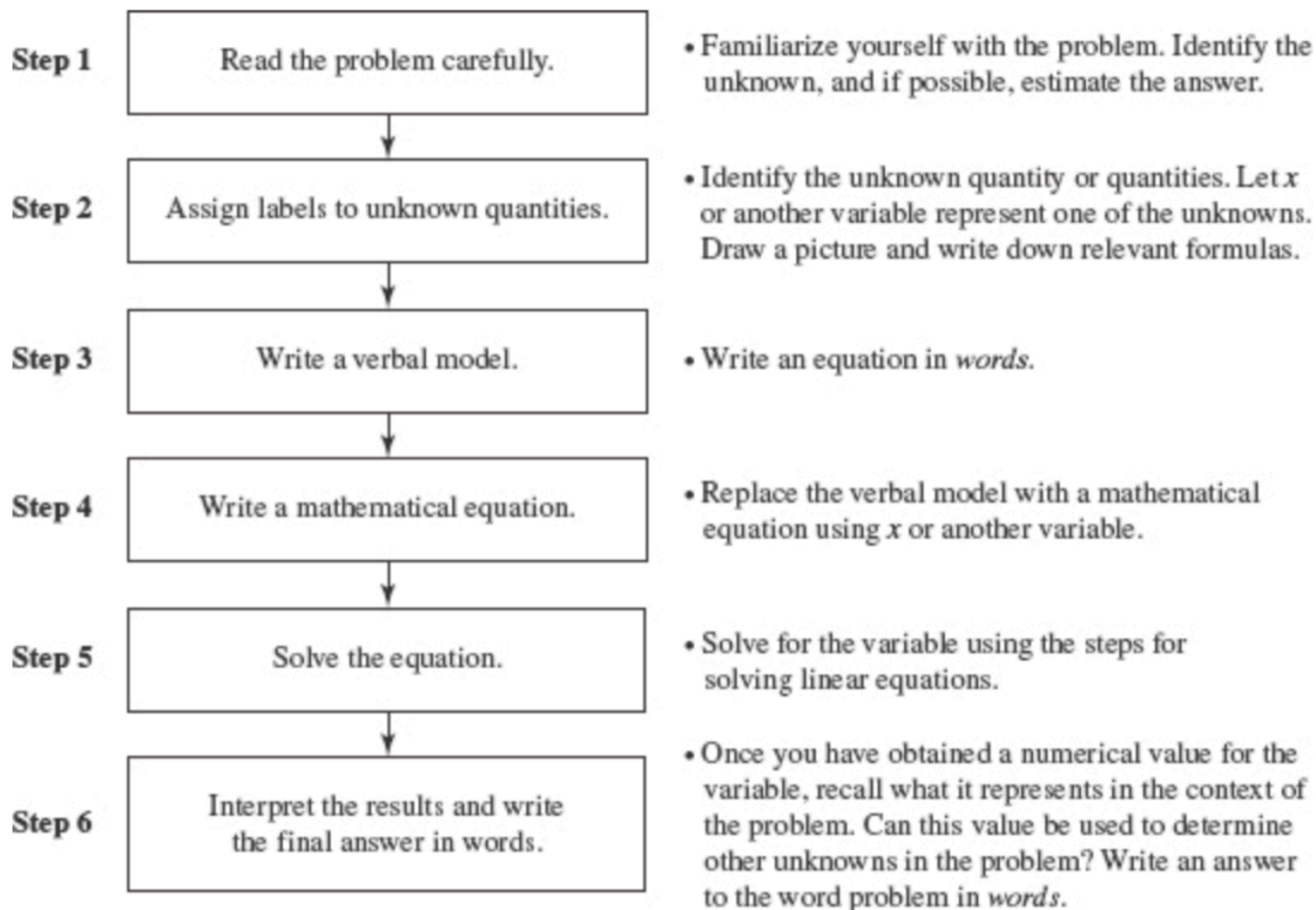


Day 13

Word problems

Problem-Solving Flowchart for Word Problems



Avoiding Mistakes

Once you have reached a solution to a word problem, verify that it is reasonable in the context of the problem.

The sum of a number and negative eleven is negative fifteen. Find the number.

Solution:

Let x represent the unknown number.

$$\begin{array}{l} \text{the sum of} \quad \quad \text{is} \\ \downarrow \quad \quad \downarrow \\ (\text{a number}) + (-11) = (-15) \\ x + (-11) = -15 \\ x + (-11) + 11 = -15 + 11 \\ x = -4 \end{array}$$

The number is -4 .

Step 1: Read the problem.

Step 2: Label the unknown.

Step 3: Write a verbal model.

Step 4: Write an equation.

Step 5: Solve the equation.

Step 6: Write the final answer in words.

Forty less than five times a number is fifty-two less than the number. Find the number.

Solution:

Let x represent the unknown number.

$$\begin{array}{ccccccc} & \text{less} & & \text{is} & & \text{less} & \\ & \downarrow & & \downarrow & & \downarrow & \\ \left(\begin{array}{c} \text{5 times} \\ \text{a number} \end{array} \right) & - & (40) & = & \left(\begin{array}{c} \text{the} \\ \text{number} \end{array} \right) & - & (52) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ 5x & - & 40 & = & x & - & 52 \end{array}$$

$$5x - 40 = x - 52$$

$$5x - x - 40 = x - x - 52$$

$$4x - 40 = -52$$

$$4x - 40 + 40 = -52 + 40$$

$$4x = -12$$

$$\frac{4x}{4} = \frac{-12}{4}$$

$$x = -3$$

The number is -3 .

Step 1: Read the problem.

Step 2: Label the unknown.

Step 3: Write a verbal model.

Step 4: Write an equation.

Step 5: Solve the equation.

Step 6: Write the final answer in words.

Avoiding Mistakes

It is important to remember that subtraction is not a commutative operation. Therefore, the order in which two real numbers are subtracted affects the outcome. The expression “forty less than five times a number” must be translated as: $5x - 40$ (not $40 - 5x$). Similarly, “fifty-two less than the number” must be translated as: $x - 52$ (not $52 - x$).

Try to solve this one on your own

In a recent Olympics, the United States won the greatest number of overall medals, followed by China. The United States won 16 more medals than China, and together they brought home a total of 192 medals. How many medals did each country win?

Solution:

In this example, we have two unknowns. The variable x can represent either quantity. However, the number of medals won by the United States is given in terms of the number won by China.

Let x represent the number of medals won by China.

Then let $x + 16$ represent the number of medals won by the United States.

$$\left(\begin{array}{c} \text{Number of} \\ \text{medals won} \\ \text{by China} \end{array} \right) + \left(\begin{array}{c} \text{Number of medals} \\ \text{won by the} \\ \text{United States} \end{array} \right) = \left(\begin{array}{c} \text{Total} \\ \text{number} \\ \text{of medals} \end{array} \right)$$

$$x + (x + 16) = 192$$

$$2x + 16 = 192$$

$$2x = 176$$

$$x = 88$$

Step 1: Read the problem.

Step 2: Label the variables.

Step 3: Write a verbal model.

Step 4: Write an equation.

Step 5: Solve the equation.

- Medals won by China, $x = 88$
- Medals won by the United States, $x + 16 = (88) + 16 = 104$

China won 88 medals and the United States won 104 medals.

What does the word percent mean?

- Percent means, “per 100.”
- So 90% means, 90 per 100 or $90/100$.
- A percent is a tool used to relate values.

Andrea spends 20% of her monthly paycheck on rent each month. If her rent payment is \$950, what is her monthly paycheck?

Solution:

Let x represent the amount of Andrea's monthly paycheck.

The problem can be rephrased as:

\$950 is 20% of what number?

↓ ↓ ↓ ↓ ↓

$$950 = 0.20 \cdot x$$

$$950 = 0.20x$$

$$\frac{950}{0.20} = \frac{0.20x}{0.20}$$

$$4750 = x$$

Andrea's monthly paycheck is \$4750.

Step 1: Read the problem.

Step 2: Label the variables.

Step 3: Write a verbal model.

Step 4: Write a mathematical equation.

Step 5: Solve the equation.

Divide both sides by 0.20.

Step 6: Interpret the results.

A new digital camera costs \$429.95.

- a.** Compute the sales tax if the tax rate is 4%.
- b.** Determine the total cost, including tax.

Solution:

- a. Let x represent the amount of tax.

$$\begin{array}{ccccc} \text{Amount} & = & (\text{percent}) & \cdot & (\text{base}) \\ \downarrow & & \downarrow & & \downarrow \\ \text{Sales tax} & = & (\text{tax rate}) & (\text{price of merchandise}) \end{array}$$

$$x = (0.04)(\$429.95)$$

$$x = \$17.198$$

$$x = \$17.20$$

The tax on the merchandise is \$17.20.

- b. The total cost is found by:

total cost = cost of merchandise + amount of tax

Therefore, the total cost is $\$429.95 + \$17.20 = \$447.15$.

Step 1: Read the problem.

Step 2: Label the variable.

Step 3: Write a verbal model.
Apply the percent equation to compute sales tax.

Step 4: Write a mathematical equation.

Step 5: Solve the equation.
Round to the nearest cent.

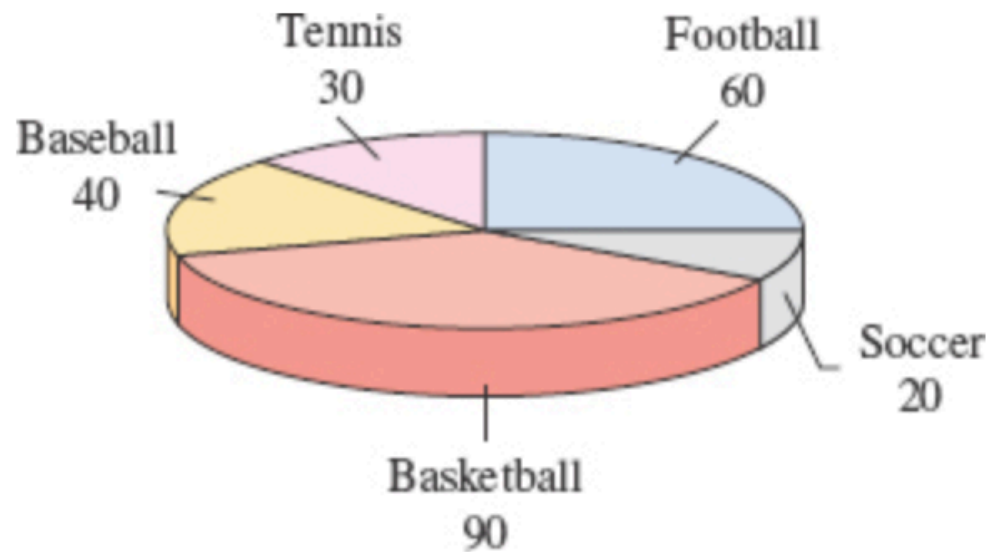
Step 6: Interpret the results.

Avoiding Mistakes

Be sure to use the decimal form of a percent within an equation.

$$4\% = 0.04$$

A group of 240 college men were asked what intramural sport they most enjoyed playing. The results are in the graph. What percent of the men surveyed preferred tennis?



Solution:

Let x represent the unknown percent (in decimal form).

The problem can be rephrased as:

$$\begin{array}{ccccccc} 30 & \text{is} & \text{what} & \text{percent} & \text{of} & 240 & ? \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ 30 & = & x & & \cdot & 240 \end{array}$$

$$30 = 240x$$

$$\frac{30}{240} = \frac{240x}{240}$$

$$0.125 = x$$

$$0.125 \times 100\% = 12.5\%$$

In this survey, 12.5% of men prefer tennis.

Step 1: Read the problem.

Step 2: Label the variable.

Step 3: Write a verbal model.

Step 4: Write a mathematical equation.

Step 5: Solve the equation.

Divide both sides by 240.

Step 6: Interpret the results.
Change the value of x to a percent form by multiplying by 100%.

Simple interest is interest that is earned or owed on principal (the original amount of money invested or borrowed). The following formula is used to compute simple interest.

$$\left(\begin{array}{c} \text{Simple} \\ \text{interest} \end{array} \right) = (\text{principal}) \left(\begin{array}{c} \text{annual} \\ \text{interest rate} \end{array} \right) \left(\begin{array}{c} \text{time} \\ \text{in years} \end{array} \right)$$

For example, to find the simple interest earned on \$2000 invested at 7.5% interest for 3 years, we have $P = \$2000$, $r = 0.075$, and $t = 3$. Thus,

$$I = Prt$$

$$\begin{aligned}\text{Interest} &= (\$2\,000)(0.075)(3) \\ &= \$450\end{aligned}$$

Jorge wants to save money to buy a car in 5 years. If Jorge needs to have \$20,250 at the end of 5 years, how much money would he need to invest in a certificate of deposit (CD) at a 2.5% interest rate?

Solution:

Let P represent the original amount invested.

$$\begin{array}{rcccl} \text{(Original)} & & & & \\ \text{(principal)} & + & \text{(interest)} & = & \text{(total)} \\ \downarrow & & \downarrow & & \downarrow \\ P & + & Prt & = & \text{total} \\ P & + & P(0.025)(5) & = & 20,250 \\ & & P + 0.125P & = & 20,250 \\ & & 1.125P & = & 20,250 \\ & & \frac{1.125P}{1.125} & = & \frac{20,250}{1.125} \\ & & P & = & 18,000 \end{array}$$

The original investment should be \$18,000.

Step 1: Read the problem.

Step 2: Label the variables.

Step 3: Write a verbal model.

Step 4: Write a mathematical equation.

Step 5: Solve the equation.

Step 6: Interpret the results and write the answer in words.

Using and Solving Formulas

Solve for the indicated variable.

a. $d = rt$ for t

b. $5x + 2y = 12$ for y

Solution:

a. $d = rt$ for t The goal is to isolate the variable t .

$$\frac{d}{r} = \frac{rt}{r}$$

Because the relationship between r and t is multiplication, we reverse the process by dividing both sides by r .

$$\frac{d}{r} = t, \text{ or equivalently } t = \frac{d}{r}$$

b. $5x + 2y = 12$ for y The goal is to solve for y .

$$5x - 5x + 2y = 12 - 5x$$

Subtract $5x$ from both sides to isolate the y term.

$$2y = -5x + 12$$

$-5x + 12$ is the same as $12 - 5x$.

$$\frac{2y}{2} = \frac{-5x + 12}{2}$$

Divide both sides by 2 to isolate y .

$$y = \frac{-5x + 12}{2}$$

Avoiding Mistakes

In the expression $\frac{-5x + 12}{2}$ do not try to divide the 2 into the 12. The divisor of 2 is dividing the entire quantity, $-5x + 12$ (not just the 12).

We may, however, apply the divisor to each term individually in the numerator. That is, $y = \frac{-5x + 12}{2}$ can be written in several different forms. Each is correct.

$$y = \frac{-5x + 12}{2} \quad \text{or} \quad y = \frac{-5x}{2} + \frac{12}{2} \Rightarrow y = -\frac{5}{2}x + 6$$

1. $A = lw$ for l

Answer

2. $-2a + 4b = 7$ for a

Answer

The formula $C = \frac{5}{9}(F - 32)$ is used to find the temperature, C , in degrees Celsius for a given temperature expressed in degrees Fahrenheit, F . Solve the formula $C = \frac{5}{9}(F - 32)$ for F .

Solution:

$$C = \frac{5}{9}(F - 32)$$

$$C = \frac{5}{9}F - \frac{5}{9} \cdot 32$$

Clear parentheses.

$$C = \frac{5}{9}F - \frac{160}{9}$$

$$\text{Multiply: } \frac{5}{9} \cdot \frac{32}{1} = \frac{160}{9}$$

$$9(C) = 9\left(\frac{5}{9}F - \frac{160}{9}\right)$$

Multiply by the LCD to clear fractions.

$$9C = \frac{9}{1} \cdot \frac{5}{9}F - \frac{9}{1} \cdot \frac{160}{9}$$

Apply the distributive property.

$$9C = 5F - 160$$

Simplify.

$$9C + 160 = 5F - 160 + 160$$

Add 160 to both sides.

$$9C + 160 = 5F$$

$$\frac{9C + 160}{5} = \frac{5F}{5}$$

Divide both sides by 5.

$$\frac{9C + 160}{5} = F$$

The answer may be written in several forms:

$$F = \frac{9C + 160}{5} \quad \text{or} \quad F = \frac{9C}{5} + \frac{160}{5} \quad \Rightarrow \quad F = \frac{9}{5}C + 32$$

Solve for the indicated variable.

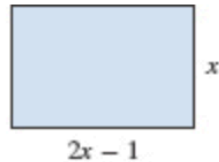
3. $y = \frac{1}{3} (x - 7)$ for x .

The length of a rectangular lot is 1 m less than twice the width. If the perimeter is 190 m, find the length and width.

Solution:

Let x represent the width of the rectangle.

Then $2x - 1$ represents the length.



$$P = 2l + 2w$$

$$190 = 2(2x - 1) + 2(x)$$

$$190 = 4x - 2 + 2x$$

$$190 = 6x - 2$$

$$192 = 6x$$

$$\frac{192}{6} = \frac{6x}{6}$$

$$32 = x$$

Step 1: Read the problem.

Step 2: Label the variables.

Step 3: Write the formula for perimeter.

Step 4: Write an equation in terms of x .

Step 5: Solve for x .

The width is $x = 32$.

The length is $2x - 1 = 2(32) - 1 = 63$.

Step 6: Interpret the results and write the answer in words.

The width of the rectangular lot is 32 m and the length is 63 m.

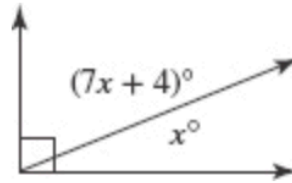
Geometry

Recall some facts about angles.

- Two angles are complementary if the sum of their measures is 90° .
- Two angles are supplementary if the sum of their measures is 180° .
- The sum of the measures of the angles within a triangle is 180° .
- The measures of vertical angles are equal.

Two complementary angles are drawn such that one angle is 4° more than seven times the other angle. Find the measure of each angle.

Solution:



Step 1: Read the problem.

Let x represent the measure of one angle.

Step 2: Label the variables.

Then $7x + 4$ represents the measure of the other angle.

The angles are complementary, so their sum must be 90° .

$$\begin{array}{ccccccc} \text{(Measure of)} & & & \text{(measure of)} & & & \\ \text{first angle} & + & & \text{second angle} & = & & 90^\circ \\ \downarrow & & & \downarrow & & \downarrow & \\ x & + & & 7x + 4 & = & & 90 \end{array}$$

Step 3: Write a verbal model.

Step 4: Write a mathematical equation.

Step 5: Solve for x .

$$\begin{aligned} 8x + 4 &= 90 \\ 8x &= 86 \\ \frac{8x}{8} &= \frac{86}{8} \\ x &= 10.75 \end{aligned}$$

One angle is $x = 10.75$.

Step 6: Interpret the results and write the answer in words.

The other angle is $7x + 4 = 7(10.75) + 4 = 79.25$.

The angles are 10.75° and 79.25° .

One angle in a triangle is twice as large as the smallest angle. The third angle is 10° more than seven times the smallest angle. Find the measure of each angle.

Solution:

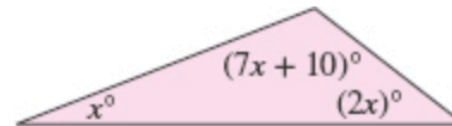
Let x represent the measure of the smallest angle.

Then $2x$ and $7x + 10$ represent the measures of the other two angles.

The sum of the angles must be 180° .

Step 1: Read the problem.

Step 2: Label the variables.



$$\left(\begin{array}{c} \text{Measure of} \\ \text{first angle} \end{array} \right) + \left(\begin{array}{c} \text{measure of} \\ \text{second angle} \end{array} \right) + \left(\begin{array}{c} \text{measure of} \\ \text{third angle} \end{array} \right) = 180^\circ$$

↓
 x

+

↓
 $2x$

+

↓
 $(7x + 10)$

$= 180$

Step 3: Write a verbal model.

Step 4: Write a mathematical equation.

Step 5: Solve for x .

Step 6: Interpret the results and write the answer in words.

$$x + 2x + 7x + 10 = 180$$

$$10x + 10 = 180$$

$$10x = 170$$

$$x = 17$$

The smallest angle is $x = 17$.

The other angles are $2x = 2(17) = 34$

$$7x + 10 = 7(17) + 10 = 129$$

The angles are 17° , 34° , and 129° .

The distance around a circular garden is 188.4 ft. Find the radius to the nearest tenth of a foot. Use 3.14 for π .

Solution:

$$C = 2\pi r \quad \text{Use the formula for the circumference of a circle.}$$

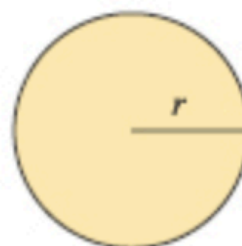
$$188.4 = 2\pi r \quad \text{Substitute 188.4 for } C.$$

$$\frac{188.4}{2\pi} = \frac{2\pi r}{2\pi} \quad \text{Divide both sides by } 2\pi.$$

$$\frac{188.4}{2\pi} = r$$

$$r \approx \frac{188.4}{2(3.14)}$$

$$= 30.0$$



$$C = 188.4 \text{ ft}$$

The radius is approximately 30.0 ft.