

Day 11

DEAR ALGEBRA,
PLEASE STOP ASKING ME
TO LOOK FOR YOUR **X**
SHE LEFT YOU AND I
DON'T KNOW **Y**

Examples

- Solve these on your own.
- Next get into groups of 2 or 3 and compare solutions.
- Then chose one person to represent your group and do the problem on the board.

$$\frac{1}{5}(z+1) + \frac{1}{4}(z+3) = 2$$

$$\frac{2}{5}x - \frac{1}{2} = \frac{7}{4} + \frac{3}{10}x$$

$$\frac{x+1}{4} + \frac{x+2}{6} = 1$$

$$\frac{25}{10}x + 3 = \frac{17}{10}x - \frac{66}{10}$$

Solve the equation by clearing decimals. $2.5x + 3 = 1.7x - 6.6$

Solution:

$$2.5x + 3 = 1.7x - 6.6$$

$$10(2.5x + 3) = 10(1.7x - 6.6)$$

Multiply both sides of the equation by 10.

$$25x + 30 = 17x - 66$$

Apply the distributive property.

$$25x - 17x + 30 = 17x - 17x - 66$$

Subtract $17x$ from both sides.

$$8x + 30 = -66$$

$$8x + 30 - 30 = -66 - 30$$

Subtract 30 from both sides.

$$8x = -96$$

$$\frac{8x}{8} = \frac{-96}{8}$$

Divide both sides by 8.

$$x = -12$$

The check is left to the reader.

The solution set is $\{-12\}$.

Solve the equation by clearing decimals. $0.2(x + 4) - 0.45(x + 9) = 12$

Solution:

$$0.2(x + 4) - 0.45(x + 9) = 12$$

$$0.2x + 0.8 - 0.45x - 4.05 = 12$$

$$100(0.2x + 0.8 - 0.45x - 4.05) = 100(12)$$

$$20x + 80 - 45x - 405 = 1200$$

$$-25x - 325 = 1200$$

$$-25x - 325 + 325 = 1200 + 325$$

$$-25x = 1525$$

$$\frac{-25x}{-25} = \frac{1525}{-25}$$

$$x = -61$$

Clear parentheses first.

Multiply both sides by 100.

Apply the distributive property.

Simplify both sides.

Add 325 to both sides.

Divide both sides by -25.

The check is left to the reader.

The solution set is $\{-61\}$.

Examples

- Solve these on your own.
- Next get into groups of 2 or 3 and compare solutions.
- Then chose one person to represent your group and do the problem on the board.

$$1.2w + 3.5 = 2.1 + w$$

$$0.25(x + 2) - 0.15(x + 3) = 4$$

$$\frac{1}{4}(3m - 4) - \frac{1}{5} = \frac{1}{4}m + \frac{3}{10}$$

$$-6.3x + 1.5 = -4.8$$

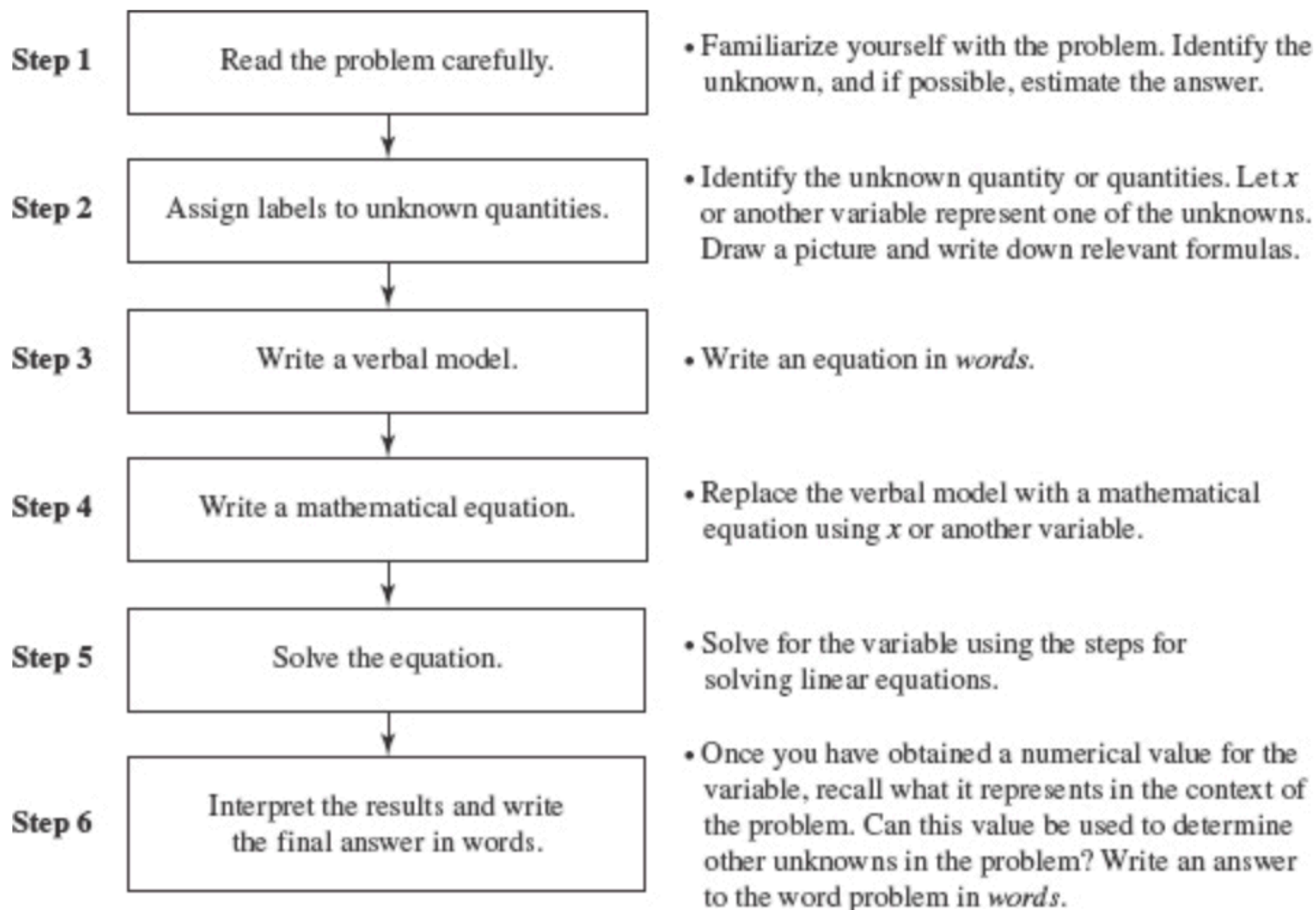
Word Problems

DEAR MATH,

I'M NOT A THERAPIST.

SOLVE YOUR OWN PROBLEMS.

Problem-Solving Flowchart for Word Problems



Avoiding Mistakes

Once you have reached a solution to a word problem, verify that it is reasonable in the context of the problem.

The sum of a number and negative eleven is negative fifteen. Find the number.

Solution:

Let x represent the unknown number.

$$\begin{array}{l} \text{the sum of} \quad \quad \text{is} \\ \downarrow \quad \quad \downarrow \\ (\text{a number}) + (-11) = (-15) \\ x + (-11) = -15 \\ x + (-11) + 11 = -15 + 11 \\ x = -4 \end{array}$$

The number is -4 .

Step 1: Read the problem.

Step 2: Label the unknown.

Step 3: Write a verbal model.

Step 4: Write an equation.

Step 5: Solve the equation.

Step 6: Write the final answer in words.

Forty less than five times a number is fifty-two less than the number. Find the number.

Solution:

Let x represent the unknown number.

$$\begin{array}{ccccccc} & \text{less} & & \text{is} & & \text{less} & \\ & \downarrow & & \downarrow & & \downarrow & \\ \left(\begin{array}{c} \text{5 times} \\ \text{a number} \end{array} \right) & - & (40) & = & \left(\begin{array}{c} \text{the} \\ \text{number} \end{array} \right) & - & (52) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ 5x & - & 40 & = & x & - & 52 \end{array}$$

$$5x - 40 = x - 52$$

$$5x - x - 40 = x - x - 52$$

$$4x - 40 = -52$$

$$4x - 40 + 40 = -52 + 40$$

$$4x = -12$$

$$\frac{4x}{4} = \frac{-12}{4}$$

$$x = -3$$

The number is -3 .

Step 1: Read the problem.

Step 2: Label the unknown.

Step 3: Write a verbal model.

Step 4: Write an equation.

Step 5: Solve the equation.

Step 6: Write the final answer in words.

Avoiding Mistakes

It is important to remember that subtraction is not a commutative operation. Therefore, the order in which two real numbers are subtracted affects the outcome. The expression “forty less than five times a number” must be translated as: $5x - 40$ (not $40 - 5x$). Similarly, “fifty-two less than the number” must be translated as: $x - 52$ (not $52 - x$).

Try to solve this one on your own

In a recent Olympics, the United States won the greatest number of overall medals, followed by China. The United States won 16 more medals than China, and together they brought home a total of 192 medals. How many medals did each country win?

Solution:

In this example, we have two unknowns. The variable x can represent either quantity. However, the number of medals won by the United States is given in terms of the number won by China.

Let x represent the number of medals won by China.

Then let $x + 16$ represent the number of medals won by the United States.

$$\left(\begin{array}{c} \text{Number of} \\ \text{medals won} \\ \text{by China} \end{array} \right) + \left(\begin{array}{c} \text{Number of medals} \\ \text{won by the} \\ \text{United States} \end{array} \right) = \left(\begin{array}{c} \text{Total} \\ \text{number} \\ \text{of medals} \end{array} \right)$$

$$x + (x + 16) = 192$$

$$2x + 16 = 192$$

$$2x = 176$$

$$x = 88$$

Step 1: Read the problem.

Step 2: Label the variables.

Step 3: Write a verbal model.

Step 4: Write an equation.

Step 5: Solve the equation.

- Medals won by China, $x = 88$
- Medals won by the United States, $x + 16 = (88) + 16 = 104$

China won 88 medals and the United States won 104 medals.

What does the word percent mean?

- Percent means, “per 100.”
- So 90% means, 90 per 100 or $90/100$.
- A percent is a tool used to relate values.

Andrea spends 20% of her monthly paycheck on rent each month. If her rent payment is \$950, what is her monthly paycheck?

Solution:

Let x represent the amount of Andrea's monthly paycheck.

The problem can be rephrased as:

\$950 is 20% of what number?

↓ ↓ ↓ ↓ ↓

$$950 = 0.20 \cdot x$$

$$950 = 0.20x$$

$$\frac{950}{0.20} = \frac{0.20x}{0.20}$$

$$4750 = x$$

Andrea's monthly paycheck is \$4750.

Step 1: Read the problem.

Step 2: Label the variables.

Step 3: Write a verbal model.

Step 4: Write a mathematical equation.

Step 5: Solve the equation.

Divide both sides by 0.20.

Step 6: Interpret the results.

A new digital camera costs \$429.95.

- a.** Compute the sales tax if the tax rate is 4%.
- b.** Determine the total cost, including tax.

Solution:

- a. Let x represent the amount of tax.

$$\begin{array}{ccccc} \text{Amount} & = & (\text{percent}) & \cdot & (\text{base}) \\ \downarrow & & \downarrow & & \downarrow \\ \text{Sales tax} & = & (\text{tax rate}) & (\text{price of merchandise}) \end{array}$$

$$x = (0.04)(\$429.95)$$

$$x = \$17.198$$

$$x = \$17.20$$

The tax on the merchandise is \$17.20.

- b. The total cost is found by:

total cost = cost of merchandise + amount of tax

Therefore, the total cost is $\$429.95 + \$17.20 = \$447.15$.

Step 1: Read the problem.

Step 2: Label the variable.

Step 3: Write a verbal model.
Apply the percent equation to compute sales tax.

Step 4: Write a mathematical equation.

Step 5: Solve the equation.
Round to the nearest cent.

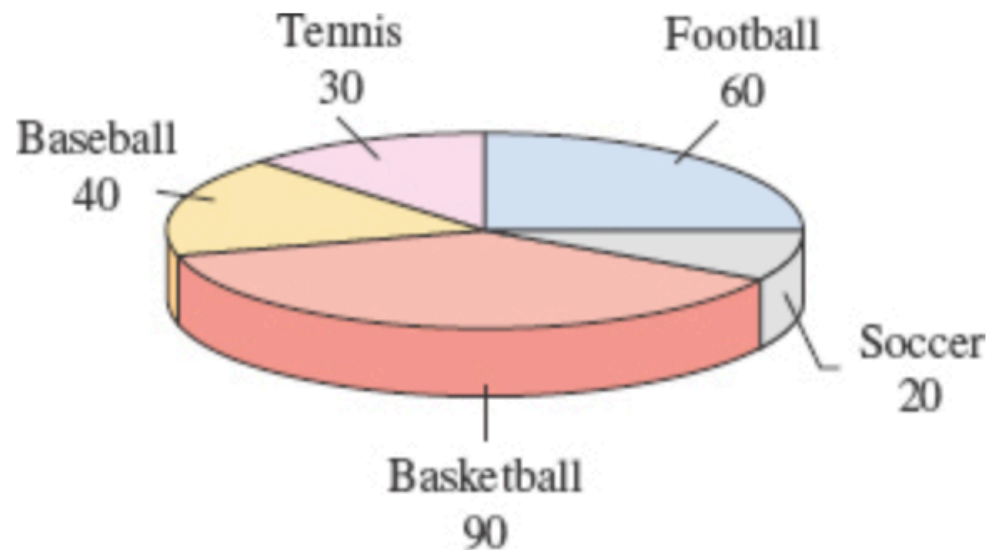
Step 6: Interpret the results.

Avoiding Mistakes

Be sure to use the decimal form of a percent within an equation.

$$4\% = 0.04$$

A group of 240 college men were asked what intramural sport they most enjoyed playing. The results are in the graph. What percent of the men surveyed preferred tennis?



Solution:

Let x represent the unknown percent (in decimal form).

The problem can be rephrased as:

$$\begin{array}{ccccccc} 30 & \text{is} & \text{what} & \text{percent} & \text{of} & 240 & ? \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ 30 & = & & x & & \cdot & 240 \end{array}$$

$$30 = 240x$$

$$\frac{30}{240} = \frac{240x}{240}$$

$$0.125 = x$$

$$0.125 \times 100\% = 12.5\%$$

In this survey, 12.5% of men prefer tennis.

Step 1: Read the problem.

Step 2: Label the variable.

Step 3: Write a verbal model.

Step 4: Write a mathematical equation.

Step 5: Solve the equation.

Divide both sides by 240.

Step 6: Interpret the results.
Change the value of x to a percent form by multiplying by 100%.

Simple interest is interest that is earned or owed on principal (the original amount of money invested or borrowed). The following formula is used to compute simple interest.

$$\left(\begin{array}{c} \text{Simple} \\ \text{interest} \end{array} \right) = (\text{principal}) \left(\begin{array}{c} \text{annual} \\ \text{interest rate} \end{array} \right) \left(\begin{array}{c} \text{time} \\ \text{in years} \end{array} \right)$$

For example, to find the simple interest earned on \$2000 invested at 7.5% interest for 3 years, we have $P = \$2000$, $r = 0.075$, and $t = 3$. Thus,

$$I = Prt$$

$$\begin{aligned}\text{Interest} &= (\$2\,000)(0.075)(3) \\ &= \$450\end{aligned}$$

Jorge wants to save money to buy a car in 5 years. If Jorge needs to have \$20,250 at the end of 5 years, how much money would he need to invest in a certificate of deposit (CD) at a 2.5% interest rate?

Solution:

Let P represent the original amount invested.

$$\begin{array}{rcccl} \text{(Original)} & & & & \\ \text{(principal)} & + & \text{(interest)} & = & \text{(total)} \\ \downarrow & & \downarrow & & \downarrow \\ P & + & Prt & = & \text{total} \\ P & + & P(0.025)(5) & = & 20,250 \\ & & P + 0.125P & = & 20,250 \\ & & 1.125P & = & 20,250 \\ & & \frac{1.125P}{1.125} & = & \frac{20,250}{1.125} \\ & & P & = & 18,000 \end{array}$$

The original investment should be \$18,000.

Step 1: Read the problem.

Step 2: Label the variables.

Step 3: Write a verbal model.

Step 4: Write a mathematical equation.

Step 5: Solve the equation.

Step 6: Interpret the results and write the answer in words.