

Day 10

Solving a Linear Equation in One Variable

Step 1 Simplify both sides of the equation.

- Clear parentheses
- Consider clearing fractions and decimals (if any are present) by multiplying both sides of the equation by a common denominator of all terms
- Combine *like* terms

Step 2 Use the addition or subtraction property of equality to collect the variable terms on one side of the equation.

Step 3 Use the addition or subtraction property of equality to collect the constant terms on the other side of the equation.

Step 4 Use the multiplication or division property of equality to make the coefficient of the variable term equal to 1.

Step 5 Check your answer.

Avoiding Mistakes

When distributing a negative number through a set of parentheses, be sure to change the signs of every term within the parentheses.

Solve the equation. $9 - (z - 3) + 4z = 4z - 5(z + 2) - 6$

Solution:

$$9 - (z - 3) + 4z = 4z - 5(z + 2) - 6$$

$$9 - z + 3 + 4z = 4z - 5z - 10 - 6$$

$$12 + 3z = -z - 16$$

$$12 + 3z + z = -z + z - 16$$

$$12 + 4z = -16$$

$$12 - 12 + 4z = -16 - 12$$

$$4z = -28$$

$$\frac{4z}{4} = \frac{-28}{4}$$

$$z = -7$$

Step 1: Clear parentheses.

Combine *like* terms.

Step 2: Add z to both sides.

Step 3: Subtract 12 from both sides.

Step 4: Divide both sides by 4 .

Step 5: The check is left for the reader.

The solution set is $\{-7\}$.

I. Conditional Equations

An equation that is true for some values of the variable but false for other values is called a **conditional equation**. The equation $x + 4 = 6$, for example, is true on the condition that $x = 2$. For other values of x , the statement $x + 4 = 6$ is false.

$$x + 4 = 6$$

$$x + 4 \text{ } -4 \text{ } = 6 \text{ } -4$$

$$x = 2 \text{ (Conditional equation) Solution set: } \{2\}$$

II. Contradictions

Some equations have no solution, such as $x + 1 = x + 2$. There is no value of x , that when increased by 1 will equal the same value increased by 2. If we try to solve the equation by subtracting x from both sides, we get the contradiction $1 = 2$. This indicates that the equation has no solution. An equation that has no solution is called a **contradiction**. The solution set is the empty set. The **empty set** is the set with no elements and is denoted by $\{ \}$.

$$x + 1 = x + 2$$

$$x - x + 1 = x - x + 2$$

$1 = 2$ (Contradiction) Solution set: $\{ \}$

Pro Tip

TIP: The empty set is also called the null set and can be expressed by the symbol \emptyset .

Avoiding Mistakes

There are two ways to express the empty set: $\{ \}$ or \emptyset . Be sure that you do not use them together. It would be incorrect to write $\{\emptyset\}$.

III. Identities

An equation that has all real numbers as its solution set is called an **identity**. For example, consider the equation, $x + 4 = x + 4$. Because the left- and right-hand sides are *identical*, any real number substituted for x will result in equal quantities on both sides. If we subtract x from both sides of the equation, we get the identity $4 = 4$. In such a case, the solution is the set of all real numbers.

$$x + 4 = x + 4$$

$$x - x + 4 = x - x + 4$$

$$4 = 4 \text{ (Identity) } \quad \text{Solution set: The set of real numbers.}$$

Solve the equation. Identify each equation as a conditional equation, a contradiction, or an identity.

a. $4k - 5 = 2(2k - 3) + 1$

b. $2(b - 4) = 2b - 7$

c. $3x + 7 = 2x - 5$

Solution:

a. $4k - 5 = 2(2k - 3) + 1$

$$4k - 5 = 4k - 6 + 1$$

Clear parentheses.

$$4k - 5 = 4k - 5$$

Combine *like* terms.

$$4k - 4k - 5 = 4k - 4k - 5$$

Subtract $4k$ from both sides.

$$-5 = -5 \quad (\text{Identity})$$

This is an identity. Solution set: The set of real numbers.

b. $2(b - 4) = 2b - 7$

$$2b - 8 = 2b - 7$$

Clear parentheses.

$$2b - 2b - 8 = 2b - 2b - 7$$

Subtract $2b$ from both sides.

$$-8 = -7 \quad (\text{Contradiction})$$

This is a contradiction. Solution set: $\{ \}$

c. $3x + 7 = 2x - 5$

$$3x - 2x + 7 = 2x - 2x - 5$$

Subtract $2x$ from both sides.

$$x + 7 = -5$$

Simplify.

$$x + 7 - 7 = -5 - 7$$

Subtract 7 from both sides.

$$x = -12 \quad (\text{Conditional equation})$$

This is a conditional equation. The solution set is $\{-12\}$. (The equation is true only on the condition that $x = -12$.)

Solve the equation by clearing fractions first. $\frac{5}{6}x - \frac{3}{4} = \frac{1}{3}$

Solution:

$$\frac{5}{6}x - \frac{3}{4} = \frac{1}{3}$$

The LCD of $\frac{5}{6}x$, $-\frac{3}{4}$, and $\frac{1}{3}$ is 12.

$$12\left(\frac{5}{6}x - \frac{3}{4}\right) = 12\left(\frac{1}{3}\right)$$

Multiply both sides of the equation by the LCD, 12.

$$\frac{\cancel{12}^2}{1}\left(\frac{5}{\cancel{6}}x\right) - \frac{\cancel{12}^3}{1}\left(\frac{3}{\cancel{4}}\right) = \frac{\cancel{12}^4}{1}\left(\frac{1}{\cancel{3}}\right)$$

Apply the distributive property (recall that $12 = \frac{12}{1}$).

$$2(5x) - 3(3) = 4(1)$$

Simplify common factors to clear the fractions.

$$10x - 9 = 4$$

$$10x - 9 + 9 = 4 + 9$$

Add 9 to both sides.

$$10x = 13$$

$$\frac{10x}{10} = \frac{13}{10}$$

Divide both sides by 10.

$$x = \frac{13}{10}$$

The solution set is $\left\{\frac{13}{10}\right\}$.

Skill Practice Solve the equation by clearing fractions.

1. $\frac{2}{5}y + \frac{1}{2} = -\frac{7}{10}$

Answer

Skill Practice Solve the equation by clearing fractions.

1. $\frac{2}{5}y + \frac{1}{2} = -\frac{7}{10}$

Answer

$\{-3\}$

Solve the equation. $\frac{1}{6}x - \frac{2}{3} = \frac{1}{5}x - 1$

Solution:

$$\frac{1}{6}x - \frac{2}{3} = \frac{1}{5}x - 1$$

$$30\left(\frac{1}{6}x - \frac{2}{3}\right) = 30\left(\frac{1}{5}x - 1\right)$$

$$\frac{\overset{5}{\cancel{30}}}{1} \cdot \frac{1}{\underset{6}{\cancel{6}}}x - \frac{\overset{10}{\cancel{30}}}{1} \cdot \frac{2}{\underset{3}{\cancel{3}}} = \frac{\overset{6}{\cancel{30}}}{1} \cdot \frac{1}{\underset{5}{\cancel{5}}}x - 30(1)$$

$$5x - 20 = 6x - 30$$

$$5x - 6x - 20 = 6x - 6x - 30$$

$$-x - 20 + 20 = -30 + 20$$

$$-x = -10$$

$$\frac{-x}{-1} = \frac{-10}{-1}$$

$$x = 10$$

The solution set is $\{10\}$.

The LCD of $\frac{1}{6}x$, $-\frac{2}{3}$, $\frac{1}{5}x$, and -1 is 30.

Multiply by the LCD, 30.

Apply the distributive property (recall $30 = \frac{30}{1}$).

Clear fractions.

Subtract $6x$ from both sides.

Add 20 to both sides.

Divide both sides by -1 .

The check is left to the reader.

Examples

- Solve these on your own.
- Next get into groups of 2 or 3 and compare solutions.
- Then choose one person to represent your group and do the problem on the board.

$$\frac{1}{5}(z+1) + \frac{1}{4}(z+3) = 2$$

$$\frac{2}{5}x - \frac{1}{2} = \frac{7}{4} + \frac{3}{10}x$$

$$\frac{x+1}{4} + \frac{x+2}{6} = 1$$

$$\frac{25}{10}x + 3 = \frac{17}{10}x - \frac{66}{10}$$

Solve the equation by clearing decimals. $2.5x + 3 = 1.7x - 6.6$

Solution:

$$2.5x + 3 = 1.7x - 6.6$$

$$10(2.5x + 3) = 10(1.7x - 6.6)$$

Multiply both sides of the equation by 10.

$$25x + 30 = 17x - 66$$

Apply the distributive property.

$$25x - 17x + 30 = 17x - 17x - 66$$

Subtract $17x$ from both sides.

$$8x + 30 = -66$$

$$8x + 30 - 30 = -66 - 30$$

Subtract 30 from both sides.

$$8x = -96$$

$$\frac{8x}{8} = \frac{-96}{8}$$

Divide both sides by 8.

$$x = -12$$

The check is left to the reader.

The solution set is $\{-12\}$.

Solve the equation by clearing decimals. $0.2(x + 4) - 0.45(x + 9) = 12$

Solution:

$$0.2(x + 4) - 0.45(x + 9) = 12$$

$$0.2x + 0.8 - 0.45x - 4.05 = 12$$

$$100(0.2x + 0.8 - 0.45x - 4.05) = 100(12)$$

$$20x + 80 - 45x - 405 = 1200$$

$$-25x - 325 = 1200$$

$$-25x - 325 + 325 = 1200 + 325$$

$$-25x = 1525$$

$$\frac{-25x}{-25} = \frac{1525}{-25}$$

$$x = -61$$

Clear parentheses first.

Multiply both sides by 100.

Apply the distributive property.

Simplify both sides.

Add 325 to both sides.

Divide both sides by -25.

The check is left to the reader.

The solution set is $\{-61\}$.

Examples

- Solve these on your own.
- Next get into groups of 2 or 3 and compare solutions.
- Then choose one person to represent your group and do the problem on the board.

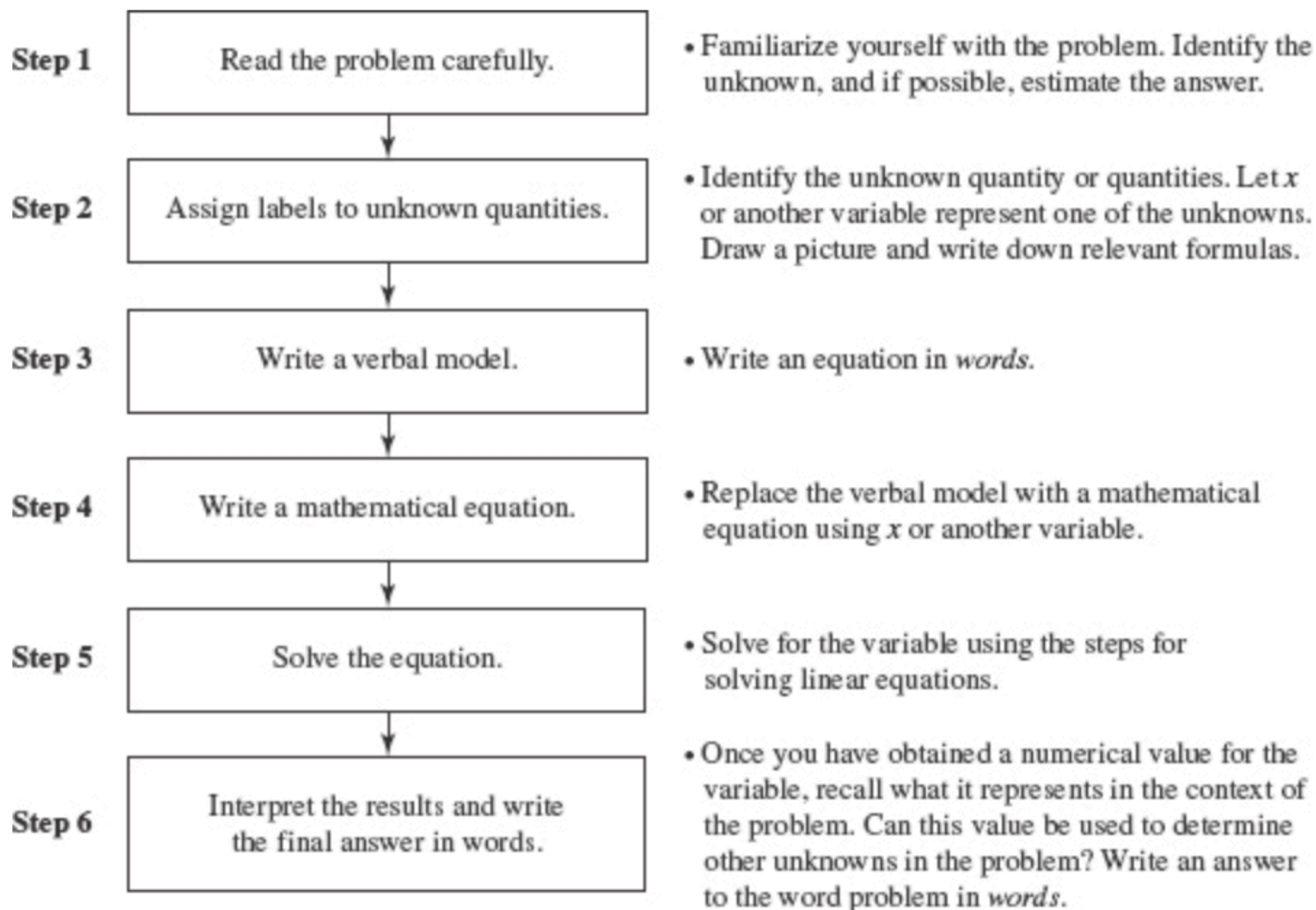
$$1.2w + 3.5 = 2.1 + w$$

$$0.25(x + 2) - 0.15(x + 3) = 4$$

$$\frac{1}{4}(3m - 4) - \frac{1}{5} = \frac{1}{4}m + \frac{3}{10}$$

$$-6.3x + 1.5 = -4.8$$

Problem-Solving Flowchart for Word Problems



Avoiding Mistakes

Once you have reached a solution to a word problem, verify that it is reasonable in the context of the problem.

The sum of a number and negative eleven is negative fifteen. Find the number.

Solution:

Let x represent the unknown number.

$$\begin{array}{l} \text{the sum of} \quad \text{is} \\ \downarrow \quad \downarrow \\ (\text{a number}) + (-11) = (-15) \\ x + (-11) = -15 \\ x + (-11) + 11 = -15 + 11 \\ x = -4 \end{array}$$

The number is -4 .

Step 1: Read the problem.

Step 2: Label the unknown.

Step 3: Write a verbal model.

Step 4: Write an equation.

Step 5: Solve the equation.

Step 6: Write the final answer in words.

Forty less than five times a number is fifty-two less than the number. Find the number.

Solution:

Let x represent the unknown number.

$$\begin{array}{ccccccc} & \text{less} & & \text{is} & & \text{less} & \\ & \downarrow & & \downarrow & & \downarrow & \\ \left(\begin{array}{c} \text{5 times} \\ \text{a number} \end{array} \right) & - & (40) & = & \left(\begin{array}{c} \text{the} \\ \text{number} \end{array} \right) & - & (52) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ 5x & - & 40 & = & x & - & 52 \end{array}$$

$$5x - 40 = x - 52$$

$$5x - x - 40 = x - x - 52$$

$$4x - 40 = -52$$

$$4x - 40 + 40 = -52 + 40$$

$$4x = -12$$

$$\frac{4x}{4} = \frac{-12}{4}$$

$$x = -3$$

The number is -3 .

Step 1: Read the problem.

Step 2: Label the unknown.

Step 3: Write a verbal model.

Step 4: Write an equation.

Step 5: Solve the equation.

Step 6: Write the final answer in words.

Avoiding Mistakes

It is important to remember that subtraction is not a commutative operation. Therefore, the order in which two real numbers are subtracted affects the outcome. The expression “forty less than five times a number” must be translated as: $5x - 40$ (not $40 - 5x$). Similarly, “fifty-two less than the number” must be translated as: $x - 52$ (not $52 - x$).