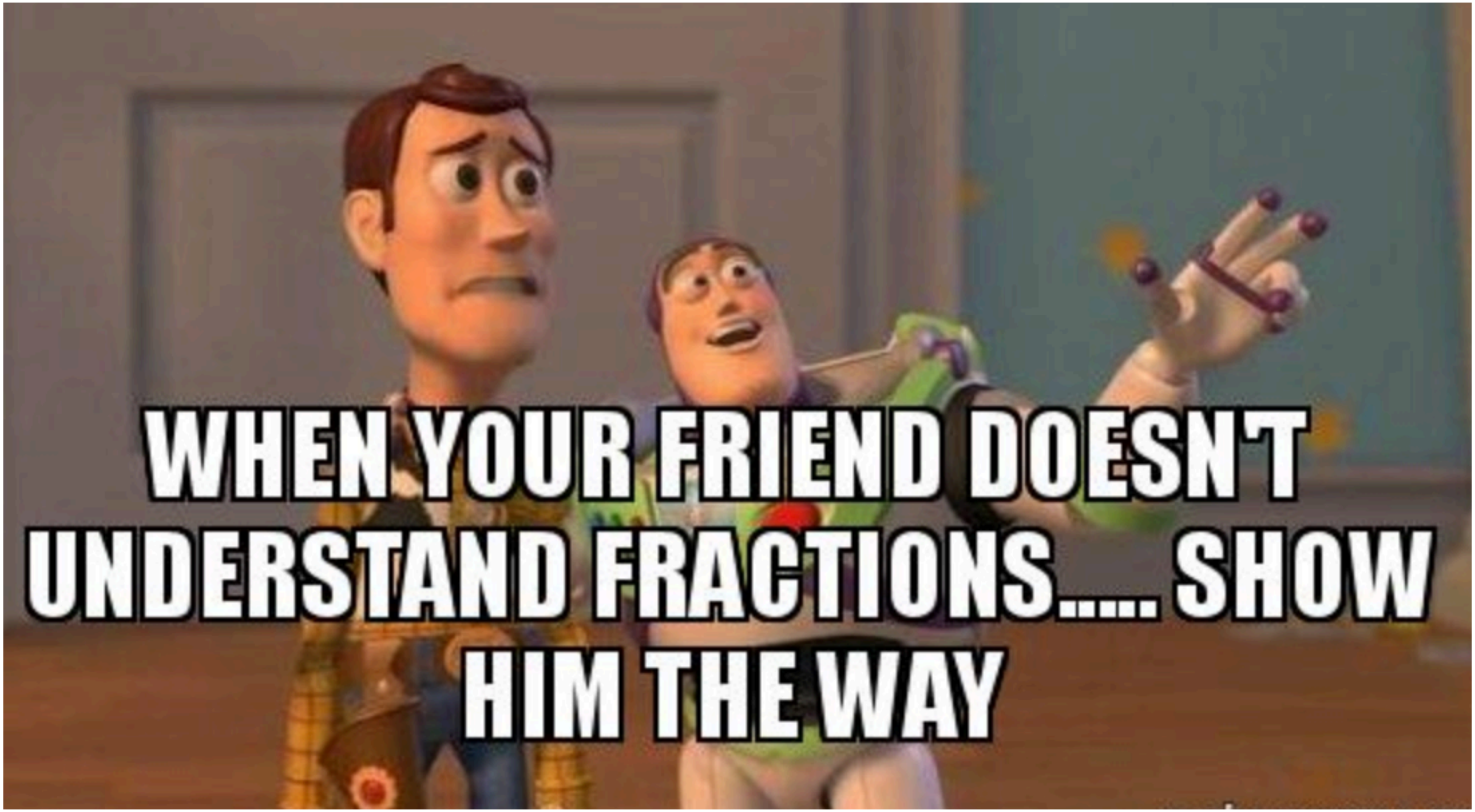
Day 3

Quick Review of Fractions



Parts of a Fraction

A Fraction and its Parts

Fractions are numbers of the form $\frac{a}{b}$, where $\frac{a}{b} = a \div b$ and b does not equal zero.

In the fraction $\frac{a}{b}$, the **numerator** is a, and the **denominator** is b.

Types of Fractions

Proper Fractions, Improper Fractions, and Mixed Numbers

1. If the numerator of a fraction is less than the denominator, the fraction is a **proper fraction**. A proper fraction represents a quantity that is less than a whole unit.
2. If the numerator of a fraction is greater than or equal to the denominator, then the fraction is an **improper fraction**. An improper fraction represents a quantity greater than or equal to a whole unit.
3. A **mixed number** is a whole number added to a proper fraction.

Fundamental Principle

Fundamental Principle of Fractions

Suppose that a number, *c*, is a common factor in the numerator and denominator of a fraction. Then

$$\frac{a \times c}{b \times c} = \frac{a}{b} \times \frac{c}{c} = \frac{a}{b} \times 1 = \frac{a}{b}$$

Multiplying Fractions

Multiplying Fractions

If b is not zero and d is not zero, then

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

To multiply fractions, multiply the numerators and multiply the denominators.

Reciprocal

The Reciprocal of a Number

Two nonzero numbers are **reciprocals** of each other if their product is 1. Therefore, the reciprocal of the fraction

$$\frac{a}{b} \text{ is } \frac{b}{a} \qquad \text{because} \qquad \frac{a}{b} \times \frac{b}{a} = 1$$

Dividing Fractions

Dividing Fractions

Let *a, b, c,* and *d* be numbers such that *b*, *c*, and *d* are not zero. Then,

$$\frac{a}{b} \div \frac{c}{d} = \underbrace{\frac{a}{b} \times \frac{d}{c}}_{\text{multiply by the reciprocal}}$$

To divide fractions, multiply the first fraction by the reciprocal of the second fraction.

Adding and Subtracting Fractions

Adding and Subtracting Fractions

Two fractions can be added or subtracted if they have a common denominator. Let *a*, *b*, and *c* be numbers such that *b* does not equal zero. Then,

$$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b} \qquad \text{and} \qquad \frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}$$

To add or subtract fractions with the same denominator, add or subtract the numerators and write the result over the common denominator.

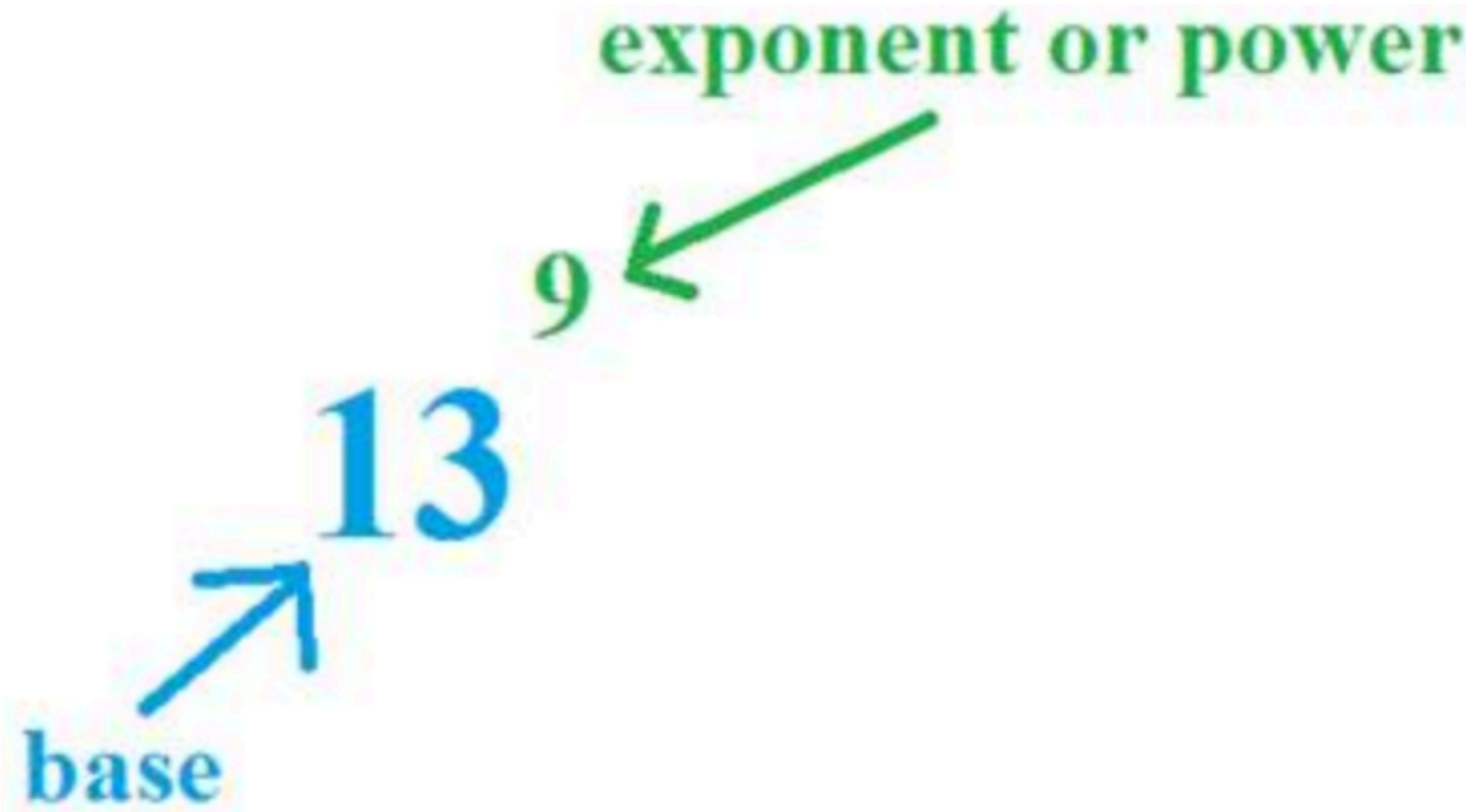
LCM

Finding the LCM of Two Numbers

Step 1 Write each number as a product of prime factors.

Step 2 The LCM is the product of unique prime factors from *both* numbers Use repeated factors the maximum number of times they appear in *either* factorization.

Exponents and the Order of Operations



Exponents

An exponent, or power, is denoted as a super script(raised number) to the right of another number, the base. It means, to multiply the base by its self, that many times.

b^p

b, is the base

p, is the power

Examples:

$x^2=x*x$

$3^3=3*3*3$

Class Examples

Try solving these:

$3^{2=}$

$2^{4=}$

$4^{2=}$

$9^{2=}$

Class Examples

Try solving these:

$3^2=9$

$2^4=16$

$4^2=16$

$2^3=8$

Question:

Why is $2^4=4^2$ but $2^3 \neq 3^2$?

Class Examples

Try solving these:

$3^2=3*3=9$

$2^5=2*2*2*2*2=32$

$5^2=5*5=25$

$9^3=9*9*9=729$

Rules of Exponents

$a^{-b}=1/(a^{b})$

$a^{b}a^{c}=a^{b+c}$

$a^{b}/a^{c}=a^{b-c}$

Examples

2^{-1}

$5^2 5^3$

$5^4/5^2$

Rules of Exponents(cont.)

$(a^b)^c=a^{bc}$

$a^{1/b}={}^b\sqrt{a}=c$

(Why is this special? Because $a=c^b$)

$(a/b)^c=a^c/b^c$

Examples

$(2^3)^2$

$25^{1/2}$

$(7/5)^2$

Class Examples

Try solving these:

$3^2 3^2$

$2^5 2^{-3}$

$25^{1/2}$

$9^{3/2}$

$(4/9)^{1/2}$

Class Examples

Try solving these:

$3^2 3^2 = 3^4 = (3*3)\ (3*3) = 9*9 = 81$

$2^5 2^{-3} = 2^{5-3} = (2*2*2*2*2)/(2*2*2) = 2^2 = 4$

$25^{1/2} = 5$

$9^{3/2} = \surd(9*9*9) = \surd(3*3*3*3*3*3) = 3*3*3 = 27$

$(4/9)^{1/2} = 2/3$

Lingo

- Please see the table on pg 39 in your ebook.

Practice

Skill Practice Translate each English phrase to an algebraic expression.

15. The product of 6 and *y*

[Answer]

16. The difference of the square root of *t* and 7

[Answer]

17. Twelve less than *x*

[Answer]

18. Twelve less *x*

[Answer]

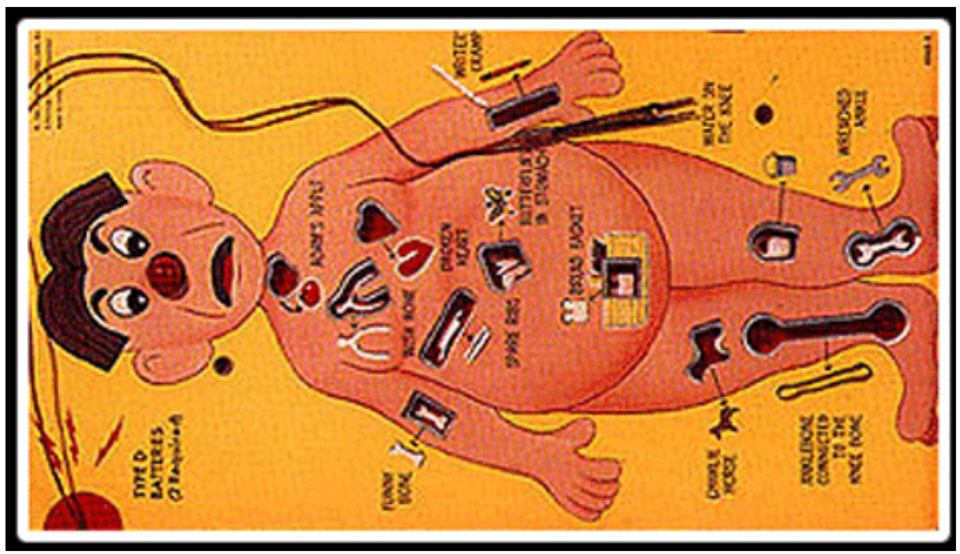
19. One more than two times *x*

[Answer]

20. Five subtracted from the absolute value of *w*

[Answer]

Orders of Operation



Not this kind… But if you are playing this kind, start with the wrench.

THE RULES!

- Pink - (Parenthesis)
- Elephants - (Exponents)
- Marching - (Multiplication)
- Down - (Division)
- A - (Addition)
- Street - (Street)

Examples

QUESTION

Evaluate the following expression.

$$[8+(14-9)\times 2]\div 6$$

EXPLANATION

We must follow the rules for the order of operations.

We start with the innermost grouping, $(14-9)$.

$$[8+(\mathbf{14-9})\times 2]\div 6$$

$$[8+\mathbf{5}\times 2]\div 6$$

We next evaluate the remaining grouping, $[8+5\times 2]$.
We do all multiplication and division before any addition or subtraction.

$$[8+\mathbf{5\times 2}]\div 6$$

$$[8+\mathbf{10}]\div 6$$

$$18\div 6$$

We then evaluate this last expression.

$$18\div 6=3$$

Order of operations with whole numbers and grouping symbols

QUESTION

Evaluate the following expression.

$$9 \times [1 + (14 + 10) \div 6]$$

EXPLANATION

We must follow the rules for the order of operations.

We start with the innermost grouping, $(14+10)$.

$$9 \times [1 + (\mathbf{14 + 10}) \div 6]$$

$$9 \times [1 + \mathbf{24} \div 6]$$

We next evaluate the remaining grouping, $[1+24 \div 6]$.
We do all multiplication and division before any addition or subtraction.

$$9 \times [1 + \mathbf{24 \div 6}]$$

$$9 \times [1 + \mathbf{4}]$$

$$9 \times 5$$

We then evaluate this last expression.

$$9 \times 5 = 45$$

QUESTION

Evaluate the following expression.

$$4 \times [2 + (20 + 22) \div 6]$$

EXPLANATION

We must follow the rules for the order of operations.

We start with the innermost grouping, $(20+22)$.

$$4 \times [2 + (\mathbf{20 + 22}) \div 6]$$

$$4 \times [2 + \mathbf{42} \div 6]$$

We next evaluate the remaining grouping, $[2+42 \div 6]$.
We do all multiplication and division before any addition or subtraction.

$$4 \times [2 + \mathbf{42 \div 6}]$$

$$4 \times [2 + \mathbf{7}]$$

$$4 \times 9$$

We then evaluate this last expression.

$$4 \times 9 = 36$$

QUESTION

Evaluate.

$$27 \div 3^2 \cdot 3$$

EXPLANATION

We must follow the rules for order of operations. Here is the order.

1. Evaluate expressions within parentheses.
2. Evaluate terms with exponents.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

If any of the rules does not apply, we skip it.

In this problem, there are no parentheses.
So, first we evaluate the exponent.

$$27 \div \mathbf{3^2} \cdot 3 = 27 \div 9 \cdot 3$$

Then, we multiply and divide from left to right.

$$\begin{aligned} \mathbf{27 \div 9} \cdot 3 &= \mathbf{3} \cdot 3 \\ &= 9 \end{aligned}$$

QUESTION

Evaluate.

$$5 \cdot \left(11 - 2^3\right)$$

EXPLANATION

We must follow the rules for order of operations. Here is the order.

1. Evaluate expressions within parentheses.
2. Evaluate terms with exponents.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

If any of the rules does not apply, we skip it.

We start by evaluating within the parentheses, $\left(11 - 2^3\right)$.
Within parentheses, we still follow the rules for order of operations.

First, we evaluate the exponent.

$$5 \cdot \left(11 - \mathbf{2^3}\right) = 5 \cdot (11 - \mathbf{8})$$

Then, we finish evaluating within the parentheses.

$$5 \cdot \mathbf{(11 - 8)} = 5 \cdot \mathbf{3}$$

Finally, we evaluate the last expression.

$$5 \cdot 3 = 15$$

Evaluate the following expression.

$$2 \times \left[2 + \left(19 + 17\right) \div 9\right]$$

$$2\times\left[\left(14+34\right)\div 8-3\right]$$

$$20 \div \left[\left(10 - 8 \right) \times 3 - 2 \right]$$

$$\left[\left(15+6\right)\div 3+1\right]\times 5$$

$$6 \times \left[2 + \left(16 - 10\right) \div 2\right]$$

$$3 \cdot \left(6+3^{3}\right)$$

$$(9-1)^2 \div 4$$

$$4 \cdot \left(2 + 2^4\right)$$

$$\frac{5}{6} + \frac{1}{3} \cdot \frac{5}{8}$$

$$\left(\frac{1}{2}+\frac{3}{5}\right)\cdot\frac{3}{8}$$